CONTROLLABILITY ANALYSIS OF MODIFIED PETLYUK STRUCTURES

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Fully thermally coupled distillation columns (Petlyuk-type columns) represent an interesting alternative to conventional distillation sequences used in multicomponent mixture separation processes, due to potential savings in both energy and capital costs. However, possible operational difficulties have limited the industrial applications of Petlyuk systems. Some of the control challenges result from the transfer of vapour stream back and forth between columns. This means that those columns do not display a uniform lower or higher pressure with respect to the other. Recently, some alternative Petlyuk-type schemes that might provide better operation properties than the traditional Petlyuk column have been proposed. In this work, the theoretical control properties of six alternative schemes to the Petlyuk system were obtained and compared. This was performed by using the singular value decomposition (SVD) technique in the frequency domain. Also, dynamic closed-loop responses for set point tracking and disturbance rejection were obtained to support the theoretical control properties. The results showed that the reduction in the number of interconnections and the use of unidirectional flows affected the dynamic properties of the complex schemes leading to potential operational advantages in thermally coupled distillation sequences.

Les colonnes de distillation totalement couplées thermiquement (colonnes de type Petlyuk) sont un bon choix par rapport aux séquences de distillation classiques utilisées dans les procédés de séparation de mélanges multicomposants en raison des économies potentielles en termes de coûts en énergie et capital. Cependant, la possibilité de difficultés opérationnelles a limité les applications industrielles des systèmes Petlyuk. Certains des défis qui se posent en matière de contrôle résultent du va-et-vient du courant de vapeur entre les colonnes. Cela signifie que ces colonnes ne présentent pas une pression basse ou haute uniforme l'une par rapport à l'autre. Récemment, des schémas différents des systèmes Petlyuk pouvant fournir de meilleures propriétés opérationnelles par rapport à la colonne traditionnelle ont été proposés. Dans ce travail, les propriétés de contrôle théorique de six nouveaux schémas du système Petlyuk ont été obtenues et comparées. Pour ce faire, on a utilisé la technique de décomposition en valeurs singulières dans le domaine des fréquences. En outre, on a obtenu des réponses en boucle fermée dynamiques pour le suivi des points de consigne et du rejet des perturbations afin de prouver les propriétés de contrôle théorique. Les résultats montrent que la réduction du nombre d'interconnexions et l'utilisation d'écoulement unidirectionnel influent sur les propriétés dynamiques des schémas complexes, offrant des avantages opérationnels potentiels dans les séquences de distillation couplées thermiquement.

Keywords: complex distillation, Petlyuk structures, control properties, singular value decomposition

INTRODUCTION

Distillation columns consume a large portion of the energy used by chemical industries. Thus, by improving energy efficiencies, important reductions in annual operating costs may be achieved. Energy savings in the range of 20–40% can be obtained in heat duties supplied to reboilers by using a three-product integrated Petlyuk column (Figure 1) instead of a conventional distillation sequence (Petlyuk et al., 1965; Tedder and Rudd, 1978; Hernández and Jiménez, 1999a). Such energy savings have promoted the development of more formally designed methods for the Petlyuk arrangement (Triantafyllou and Smith, 1992; Dunnebier and Pantelides, 1999; Halvorsen and Skogestad, 1999; Amminudin et al., 2001; Kim, 2002; Muralikrishna et al., 2002; Kim, 2003). Even though a great deal of information related to potential energy savings has been available during the past two decades for this type of systems, there is still some reluctance by the industry to use these complex

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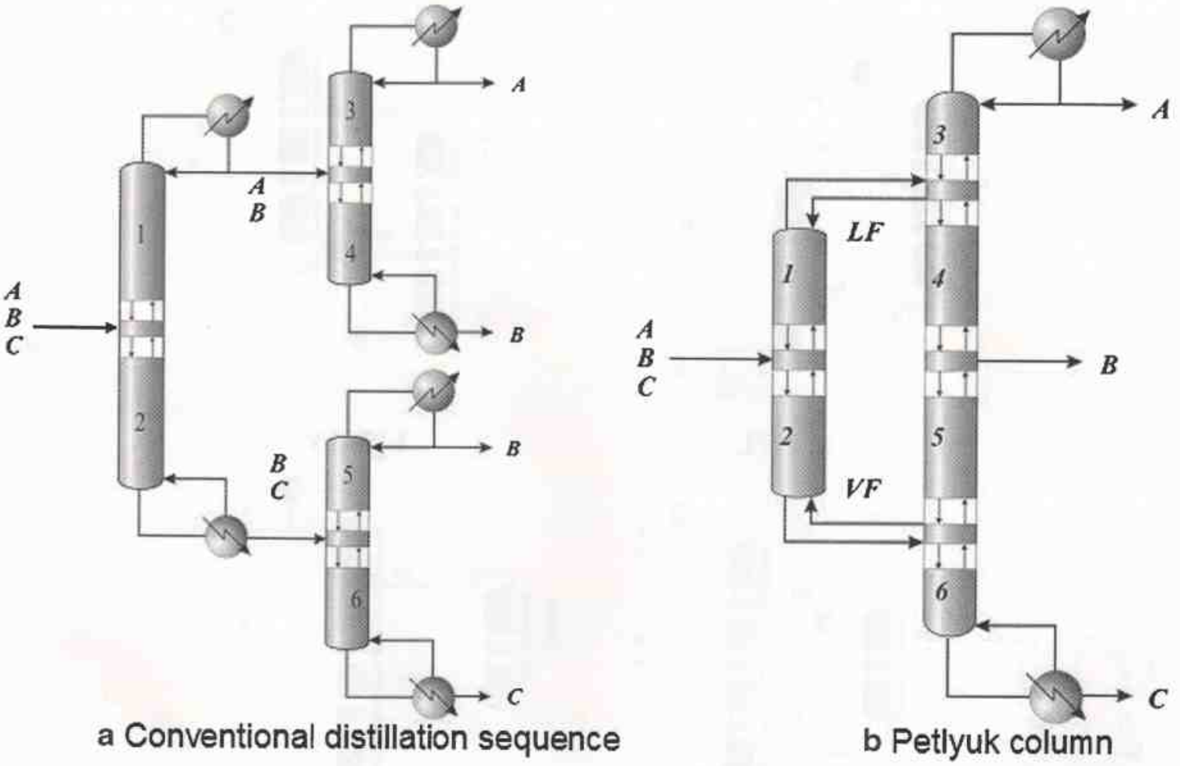
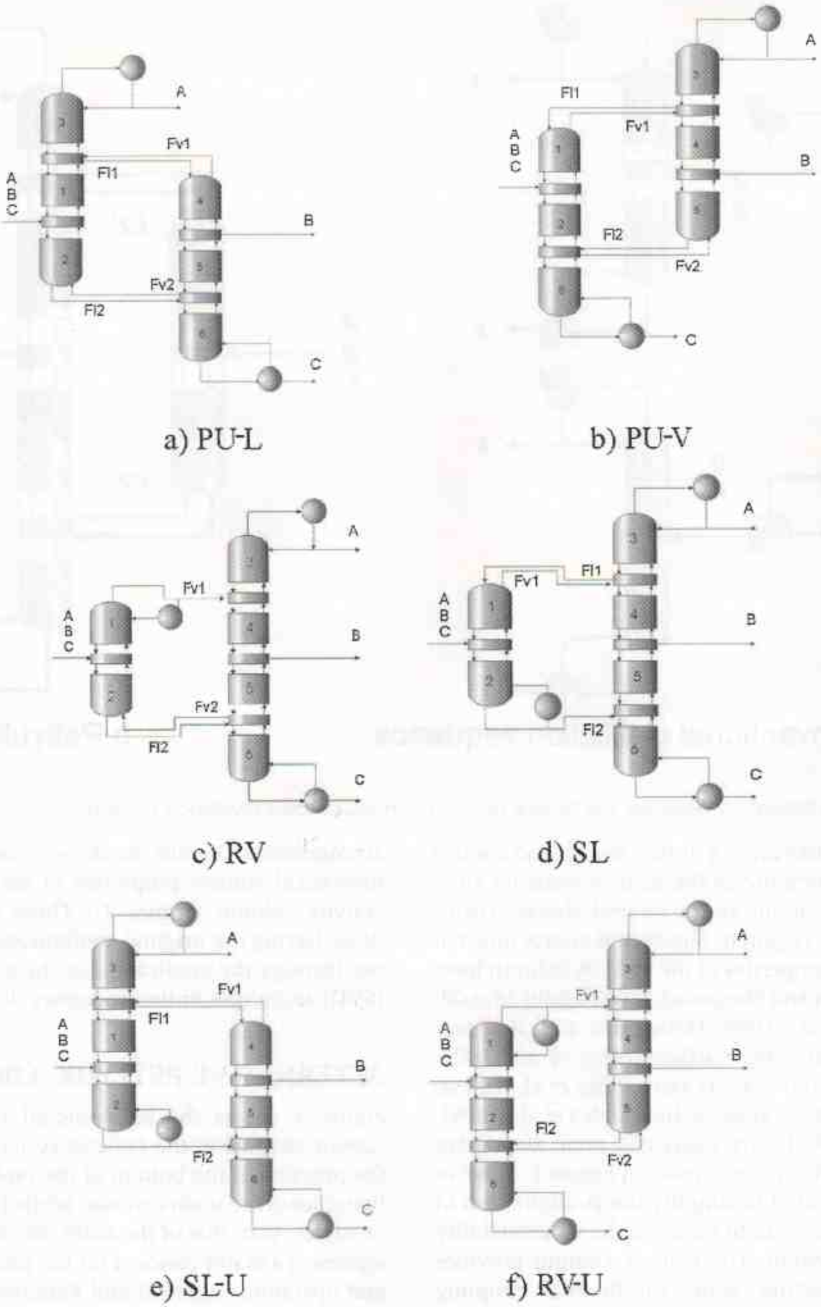


Figure 1. Ternary conventional distillation sequence and Petlyuk or fully thermally coupled distillation column.

integrated columns. Difficulties arising during design and control have been reported in the literature as the main reasons for such reluctance. A better understanding of the control characteristics of these systems is therefore required. Significant efforts directed to understand the dynamic properties of the Petlyuk column have been recently reported (Wolff and Skogestad, 1995; Abdul-Mutalib and Smith, 1998; Serra et al., 1999; Hernández and Jiménez, 1999b; Jiménez et al., 2001; Segovia-Hernández et al., 2002; Segovia-Hernández et al., 2004; Segovia-Hernández et al., 2005a; Segovia-Hernández et al., 2005b; Segovia-Hernández et al., 2006). Agrawal and Fidkowski (1998) have suggested some alternative options and extensions for the system shown in Figure 1. Some of those modifications are intended to simplify the configuration of the original Petlyuk scheme so as to improve the controllability properties of the original system. The Petlyuk column provides a fully interconnected structure with two thermal coupling interconnections resulting in vapour flowing back and forth between the columns. This is a design challenge on the system, since columns cannot be designed at a uniformly higher pressure with respect to the others. Since the Petlyuk structures typically provide the highest energy savings from complex systems, a special interest to provide modified configurations to such arrangement has arisen. Another important reason is that Petlyuk structures have been implemented in the chemical industry by using a dividing-wall distillation column that causes reductions in both energy and capital costs (Kaibel and Schoenmarkers, 2002). Some of the new structures that have been suggested to improve the expected control properties of the Petlyuk system have recently been analyzed in terms of their potential energy requirements. It has been found in many cases that they provide similar energy savings (Jiménez et al., 2003) and thermodynamic efficiencies (Hernández et al., 2006) as the original system. Hence, the next step is to determine if the original dynamic properties of the Petlyuk column can be improved in the proposed complex arrangements. In this work, we conducted an analysis on the theoretical control properties of six alternative schemes to the Petlyuk column (Figure 2). These schemes were compared to those having the original configuration. The analysis was carried out through the application of the singular value decomposition (SVD) technique in the frequency domain.

ALTERNATIVE PETLYUK–LIKE CONFIGURATIONS

Figure 1 shows the bidirectional flow of the interconnecting vapour streams in the Petlyuk system. To achieve these streams, the pressure at the bottom of the prefractionator should be lower than that of the main column, while the pressure at the top should be higher than that of the main column. These requirements may represent a major concern for the proper Petlyuk implementation and operation. Agrawal and Fidkowski (1998) have studied this problem and they suggested some conceptual modifications to the Petlyuk configuration that might improve its operational properties. One way to do this is by using unidirectional flows between the columns. Another way is by reducing the number of interconnections, thus providing simpler arrangements (and therefore systems with better control perspectives). Figure 2 shows six alternative structures to the Petlyuk column that, in principle, could provide more operable systems. The new design topologies are directly derived from the original arrangement. As Figure 1 shows, the six sections of the Petlyuk system provided the basis for the structures of the new systems. The corresponding sections are identified in Figure 2. For instance, the system in Figure 2a was obtained by moving section 3 along with the condenser to the top of the first column in the Petlyuk system. The resulting structure provides two liquid streams that flow unidirectionally from the first to the second column. This configuration is identified in this work as a Petlyuk arrangement with unidirectional liquid flows (PUL). If section 6 of the Petlyuk column and the reboiler



a) PU-L b) PU-V c) RV d) SL e) SL-U f) RV-U

Figure 2. Alternative Petlyuk-like schemes.

are removed and placed at the bottom of the first column, a Petlyuk arrangement with unidirectional vapour flow going from the first to the second column is obtained (PUV, Figure 2b). The other four new arrangements are obtained through a reduction in the number of interconnections of the original Petlyuk scheme. To accomplish that, the addition of a condenser or a reboiler in the prefractionator is required. Figure 2c and d show two modifications of the original Petlyuk configuration. In both cases, a reduction in one interconnecting stream resulted in structures with vapour (RV) or liquid (SL) interconnections. Partially coupled structures with liquid (SLU) or vapour (RVU) interconnections are shown in Figure 2e and f; such structures resulted from the elimination of one interconnection to the fully coupled arrangements of Figure 2a and b. Jiménez et al. (2003) have reported a study on the design and optimization of the operating conditions to achieve minimum energy consumption for the alternate schemes. On the other hand, Hernández et al. (2006) have found that such new systems can provide similar second law efficiencies and energy consumptions as the original Petlyuk system. Although the new design structures seem to favour the dynamic properties, no previous studies on this matter have been reported.

DESIGN OF THE COMPLEX SCHEMES

In the first part of the study base designs for the Petlyuk system and for each of the six alternative structures were obtained. For the Petlyuk column, the method reported by Hernández and Jiménez (1999a) was used. A base design was obtained from the tray structure of a conventional distillation sequence consisting of a prefractionator followed by two binary columns. In the first column of the conventional distillation column, the lightest component (A) is separated from the heaviest component (C), and then two binary columns are required for the splits A/B and B/C, respectively. The conventional distillation sequence is used to obtain the tray structure of the Petlyuk column, the number of stages in the prefractionator is set equivalent to that of the first distillation column of conventional distillation sequence and the number of stages in the main distillation column are equal to the sum of stages of the last two binary distillation column (see Figure 1). After a section analogy procedure was used to provide the tray arrangement of the Petlyuk system, an exploration was conducted on the interconnecting streams (LF and VF) to detect the values that resulted in a minimum of energy consumption in the Petlyuk system. The design (tray structure) of the Petlyuk system served as the basis for obtaining the designs of the six alternative schemes. The corresponding tray structure analogy for each new arrangement is shown in Figure 2 with respect to the sections of Figure 1. To provide a consistent comparison of the alternate schemes with the Petlyuk column, the remaining degrees of freedom for each alternate arrangement were also used to detect the operating conditions that provided the minimum energy consumption for every case. More details of the procedure and values of the design variables for the proposed schemes are available in Jiménez et al. (2003).

CONTROL PROPERTIES

In the second part of the study open loop dynamic responses to set point changes around the assumed operating point (which corresponds to that with minimum energy consumption for each configuration) were obtained. The dynamic responses were obtained through the use of Aspen Dynamics 11.1™. Transfer function matrices (G) were then obtained for each case, and they were subjected to SVD:

$$G = V\Sigma W^{H} \qquad (1)$$

where $\Sigma = \text{diag}\ (\sigma_1, \ldots, \sigma_n)$, σ_i = singular value of $G = \lambda_i^{1/2}$; $V = (v_1, v_2, \ldots)$ matrix of left singular vectors, and $W = (w_1, w_2, \ldots)$ matrix of right singular vectors. Two parameters of interest are the minimum singular value, σ_*, and the ratio maximum (σ^*) to minimum singular values or condition number:

$$\gamma^* = \sigma^* / \sigma_* \qquad (2)$$

According to previous works (Klema and Laub, 1980; Lau et al., 1985; Chen et al., 1994; Hernández and Jiménez, 1999b; Jiménez et al., 2001), the minimum singular value is a measure of the invertibility of the system and, therefore, represents a measure of potential problems of the system under the action of feedback controllers. The condition number represents the sensitivity of the system under uncertainties in process parameters and modelling errors. From a physical point of view, low values of the minimum singular value and high values of the condition number imply large movements in the control valves for changes in the set points and load rejection.

Finally in this work, the SVD is computed considering perfect control (Morari, 1983). This is achieved when the output is identically equal to the reference, i.e., $y = 0$.

CASE STUDIES

The case studies for this research were selected in an attempt to represent different separation difficulties and different intermediate component contents in ternary mixtures. Three mixtures with different ease of separability index values (ESI, the ratio of relative volatilities of the split AB to the split BC, as defined by Tedder and Rudd, 1978) were considered. The components of the selected mixtures are indicated in Table 1. To examine the effect of the intermediate component composition on the control properties, two types of feed compositions were assumed. One feed was with a low content of the intermediate component (where mole fractions of A, B, C, were equal to 0.40, 0.20, 0.40, feed F1) and another was with a high content of the intermediate component (A, B, C equal to 0.15, 0.70, 0.15, feed F2) were used. The total feed flow rate for all cases was of 45.5 kmol/h. Product purities of 98.7, 98, and 98.6 mol% for A, B, and C, respectively, were assumed as part of the design specifications. Since the feed was composed of a hydrocarbon mixture, the Chao-Seader correlation was used to predict the thermodynamic properties. The design pressure for each sequence was chosen in such a way that all the condensers could be operated with cooling water. In this work, for each case study the steady-state design variables reported by Jiménez et al. (2003) were used. Figure 3 shows the minimization of energy consumption for the Petlyuk column to obtain the steady-state design. During the determination of the optimum energy consumption, three design specifications for product purities A, B, and C were set to avoid deviations to the required purities. Using the case of M1F1 as a representative one, Table 2 contains the design variables of the seven distillation arrangements.

Table 1. Mixtures analyzed

Mixture	Components	ESI
M1	*n*-Pentane *n*-Hexane *n*-Heptane	1.04
M2	*n*-Butane Isopentane *n*-Pentane	1.86
M3	Isobutane *n*-Butane *n*-Hexane	0.18

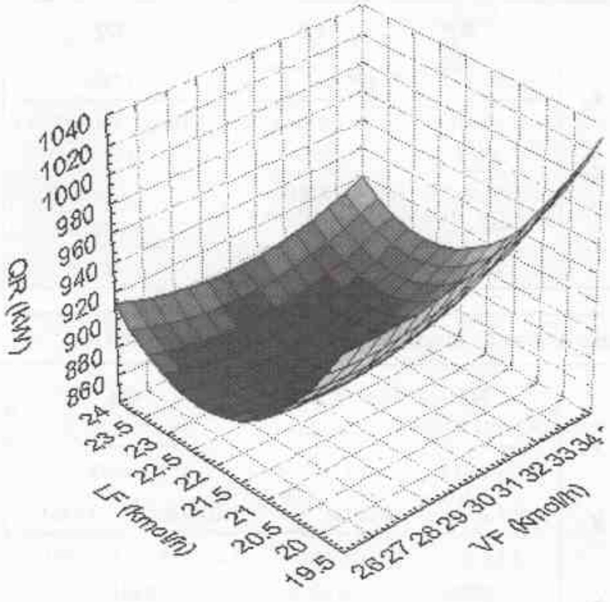


Figure 3. Search for the minimum energy consumption of the Petlyuk scheme.

Table 2. Important design variables for the seven complex distillation arrangements for M1F1

Tray sections	Section 1 = 9, section 2 = 8, section 3 = 9, section 4 = 8, section, 5 = 10, section 6 = 9
Petlyuk column	LF = 15.8757 kmol/h, VF = 38.5554 kmol/h Reflux ratio = 2.4849, reboiler duty = 488.4016 kW
PU-L sequence	FL1 = 24.9476 kmol/h, FV2 = 40.8233 kmol/h Reflux ratio 1 = 2.5494, reboiler duty 2 = 496.3197 kW
PU-V sequence	FV1 = 18.5973 kmol/h, FL2 = 18.5973 kmol/h Reflux ratio 2 = 2.5094, reboiler duty 1 = 491.0531 kW
RV sequence	FV1 = 21.3642 kmol/h, FV2 = 36.7863 kmol/h Reflux ratio 1 = 0.6752, reflux ratio 2 = 1.6353 Reboiler duty 2 = 493.9371 kW
SL sequence	FL1 = 15.6035 kmol/h, FL2 = 22.7703 kmol/h Reflux ratio 2 = 3.3683, reboiler duty 1 = 325.0398 kW Reboiler duty 2 = 279.8008 kW
SL-U sequence	FL1 = 41.7305 kmol/h, FL2 = 23.5868 kmol/h Reflux ratio 1 = 3.4441, reboiler duty 1 = 306.5675 kW Reboiler duty 2 = 312.0427 kW
RV-U sequence	FV1 = 21.5909 kmol/h, FV2 = 21.3188 kmol/h Reflux ratio 1 = 0.7066, reboiler duty 1 = 486.7177 kW Reflux ratio 2 = 1.5569

RESULTS

In the first part of the study, the theoretical control properties of the complex distillation sequences were obtained and associated to the manipulated and controlled variables. In the second part of the study, the complex distillation sequences were studied under the action of three proportional integral controllers for changes in set points or disturbances in the feed.

The SVD technique requires transfer function matrices which are generated by implementing step changes in the manipulated variables of the optimum design distillation sequences and by registering the dynamic responses of the three products. It is important to note that step tests excite the plant in the low frequency region. Moreover, this simple identification technique will most likely give meaningless results if there are large relative gains within the bandwidth where the model is going to be used (Kaymak and Luyben, 2005; Skogestad and Postlethwaite, 2005). For the distillation sequences presented in this work, three controlled variables were considered, the product composition A, B, C (x_A, x_B, x_C). Similarly, three manipulated variables were defined, the reflux ratio (R), the heat duties supplied to the reboilers (C), and the third variable was the side stream flow rate (L). Also, perfect control level in the reflux drum and the bottom accumulator of the column was assumed (with distillate and bottoms flow rates as the corresponding manipulated variables). Singular values depend on the units of the variables; therefore, the scaling of the gains is necessary. In this work, the controlled variables are bounded between 0 and 1 and the changes in the manipulated variables were associated to the fraction in the opening of the control valve (bounded between 0 and 1). After the optimum designs were obtained, open-loop dynamic simulations

Table 3. Transfer function matrix for Petlyuk column (M1F1)

$$\begin{array}{c} \\ X_A \\[2ex] X_B \\[2ex] X_C \end{array} \begin{array}{c} \begin{array}{ccc} \qquad \mathbf{R2} \qquad & \qquad \mathbf{L2} \qquad & \qquad \mathbf{C2} \qquad \end{array} \\ \left[\begin{array}{ccc} \dfrac{0.0072\,e^{-0.04\,s}}{1+0.987\,s} & \dfrac{0.0072\,e^{-0.04\,s}}{1+0.987\,s} & -\dfrac{4.7272}{0.1141\,s^2+0.6437\,s+1} \\ \dfrac{0.012}{1-0.2842\,s} & \dfrac{0.012}{1+0.2842\,s} & -\dfrac{2.6172}{1+3.2145\,s} \\ -\dfrac{0.0036\,e^{-0.55\,s}}{1+4.0525\,s} & -\dfrac{0.0028\,e^{-0.26\,s}}{1+2.51\,s} & \dfrac{3.3556}{1+2.1234\,s} \end{array} \right] \end{array}$$

Table 4. Transfer function matrix for PU-V scheme (M1F1)

$$\begin{array}{c} \\ X_A \\[2ex] X_B \\[2ex] X_C \end{array} \begin{array}{c} \begin{array}{ccc} \qquad \mathbf{R2} \qquad & \qquad \mathbf{L2} \qquad & \qquad \mathbf{C1} \qquad \end{array} \\ \left[\begin{array}{ccc} \dfrac{0.0136}{1+0.81\,s} & \dfrac{0.0136}{1+0.81\,s} & -\dfrac{4.4328}{1+0.658\,s} \\ \dfrac{0.004\,e^{-0.69\,s}}{1+9.81\,s} & \dfrac{0.004\,e^{-0.69\,s}}{1+9.81\,s} & \dfrac{0.1608}{1+0.142\,s}-\dfrac{3.7563}{1+1.7025\,s} \\ -\dfrac{0.0096}{1+3.21\,s} & -\dfrac{0.0096}{1+3.21\,s} & \dfrac{2.4412}{1+1.49\,s} \end{array} \right] \end{array}$$

Table 5. Transfer function matrix for PU-L scheme (M1F1)

$$\begin{array}{c} \\ X_A \\[2ex] X_B \\[2ex] X_C \end{array} \begin{array}{c} \begin{array}{ccc} \qquad \mathbf{R1} \qquad & \qquad \mathbf{L2} \qquad & \qquad \mathbf{C2} \qquad \end{array} \\ \left[\begin{array}{ccc} \dfrac{0.002}{0.0071\,s^2+0.0709\,s+1} & \dfrac{0.002}{0.0071\,s^2+0.0709\,s+1} & -\dfrac{4.4868}{1+0.63\,s} \\ \dfrac{0.0044\,e^{-0.17\,s}}{1+3.144\,s} & \dfrac{0.0044\,e^{-0.17\,s}}{1+3.144\,s} & -\dfrac{2.572}{(1+2.25\,s)^2} \\ \dfrac{0.0024}{1+0.282\,s}-\dfrac{0.01}{1+5.0881\,s} & \dfrac{0.0024}{1+0.146\,s}-\dfrac{0.01}{1+5.94\,s} & \dfrac{3.1008}{1+2.15\,s} \end{array} \right] \end{array}$$

Table 6. Transfer function matrix for RV scheme (M1F1)

$$\begin{array}{c} \\ x_A \\[2ex] x_B \\[2ex] x_C \end{array} \begin{array}{c} \begin{array}{ccc} \qquad \mathbf{R2} \qquad & \qquad \mathbf{L2} \qquad & \qquad \mathbf{C2} \qquad \end{array} \\ \left[\begin{array}{ccc} \dfrac{0.0216}{1+0.85\,s}-\dfrac{0.0172}{1+2.698\,s} & \dfrac{0.0216}{1+0.8\,s}-\dfrac{0.0172}{1+2.8\,s} & -\dfrac{5.0936}{0.0796\,s^2+0.742\,s+1} \\ \dfrac{0.0084}{1+3.288\,s} & \dfrac{0.0084}{1+3.275\,s} & -\dfrac{5.8888}{0.2249\,s^2+1.8952\,s+1} \\ \dfrac{0.008}{1+0.35\,s}-\dfrac{0.006}{1+2\,s} & \dfrac{0.0104}{1+0.56\,s}-\dfrac{0.0084}{1+2.1\,s} & \dfrac{2.0276}{1+1.1966\,s} \end{array} \right] \end{array}$$

Table 7. Transfer function matrix for SL scheme (M1F1)

$$\begin{array}{cc} & \begin{matrix} \textbf{R2} & \qquad\qquad\qquad\qquad \textbf{L2} & \qquad\qquad\qquad \textbf{C2} \end{matrix} \\ \begin{matrix} X_A \\ \\ X_B \\ \\ X_C \end{matrix} & \begin{bmatrix} -\dfrac{0.0064\,e^{-0.06\,s}}{1+1.048\,s} & -\dfrac{0.0064\,e^{-0.06\,s}}{1+1.048\,s} & -\dfrac{2.478}{1+0.828\,s} \\ -\dfrac{0.0204}{1+1.6111\,s} & -\dfrac{0.0204}{1+1.7883\,s} & -\dfrac{2.8268}{1+1.2642\,s} \\ -\dfrac{0.0112\,e^{-0.02\,s}}{1+0.45\,s}+\dfrac{0.0068}{1+1.25\,s} & -\dfrac{0.0112\,e^{-0.02\,s}}{1+0.45\,s}+\dfrac{0.0068}{1+1.25\,s} & \dfrac{0.0023}{1+0.9045\,s} \end{bmatrix} \end{array}$$

Table 8. Transfer function matrix for RV-U scheme (M1F1)

$$\begin{array}{cc} & \begin{matrix} \textbf{R1} & \qquad\qquad\qquad\qquad \textbf{L2} & \qquad\qquad\qquad \textbf{C2} \end{matrix} \\ \begin{matrix} X_A \\ \\ X_B \\ \\ X_C \end{matrix} & \begin{bmatrix} \dfrac{0.0108}{0.2406\,s^2+0.5135\,s+1} & \dfrac{0.0116}{0.3123\,s^2+0.7122\,s+1} & -\dfrac{4.9108}{0.1499\,s^2-0.6196\,s+1} \\ \dfrac{0.0084\,e^{-0.2\,s}}{1-6.1033\,s} & \dfrac{0.0084}{1+5.1737\,s} & \dfrac{0.1292}{1+0.0529\,s}-\dfrac{4.8448}{1+2.4173\,s} \\ -\dfrac{0.0116}{1+2.4748\,s} & -\dfrac{0.0144\,e^{-0.06\,s}}{1+4.5367\,s} & \dfrac{2.2656}{1+1.3417\,s} \end{bmatrix} \end{array}$$

Table 9. Transfer function matrix for SL-U scheme (M1F1)

$$\begin{array}{cc} & \begin{matrix} \textbf{R1} & \qquad\qquad \textbf{L2} & \qquad\qquad \textbf{C2} \end{matrix} \\ \begin{matrix} X_A \\ \\ X_B \\ \\ X_C \end{matrix} & \begin{bmatrix} -\dfrac{0.0592}{1+0.5017\,s} & -\dfrac{0.0592}{1+0.5028\,s} & -\dfrac{2.954}{1+0.3411\,s} \\ -\dfrac{0.0336}{1+0.4993\,s} & -\dfrac{0.0336}{1+0.5073\,s} & -\dfrac{2.9704}{1+0.8556\,s} \\ \dfrac{0.0252}{1+0.8777\,s} & \dfrac{0.0256}{1+0.8777\,s} & \dfrac{1.3424}{1+0.5321\,s} \end{bmatrix} \end{array}$$

were obtained in Aspen Dynamics 11.1™ to determine the transfer function matrices. For the case study M1F1, Tables 3 to 9 show the transfer function matrices generated by using step changes in the manipulated variables and recording the dynamic behaviour of the three product compositions (A, B, and C). The transfer function matrix shown in Table 3 corresponds to the Petlyuk arrangement. As this table shows, the dynamic responses can be adjusted to first and second order models either with or without dead times. A similar transfer function matrix can be obtained for alternate schemes (Tables 4 to 9) and for all cases studies.

Figures 4 and 5 present the results for the M1F1 case of study. These figures show that the SL scheme presents lower condition numbers and higher values of the minimum singular value over the whole frequency range. It should be expected that the SL system exhibits better control properties than the other sequence under feedback control and that is better conditioned to resist the effect of disturbances in comparison with other distillation schemes. PUL and PUV schemes show the worst results in the whole frequency range. The Petlyuk column presents a good dynamic behaviour in comparison with the SL scheme. In relation to the case M1F2, Figures 6 and 7 show that, at low frequencies, all arrangements but the PUL configuration exhibit lower values of condition number. However, as the frequency increases, the condition number drastically increases. In the case of the minimum singular value, similar results were obtained. In general, it can be stated that the Petlyuk column offers better conditioning properties regarding model uncertainties, as compared to the other arrangements presented in this study. As a result, small movements in the control variables can be expected for the Petlyuk distillation column in contrast to those expected in the other complex distillation sequences.

For the case M2F1, a clear tendency was observed. The PUV and RVU schemes showed the highest values for the condition number and the lowest σ_* in the whole frequency range. A detailed analysis shows that the SL and RV options are therefore better conditioned. Similarly, the Petlyuk column exhibited good

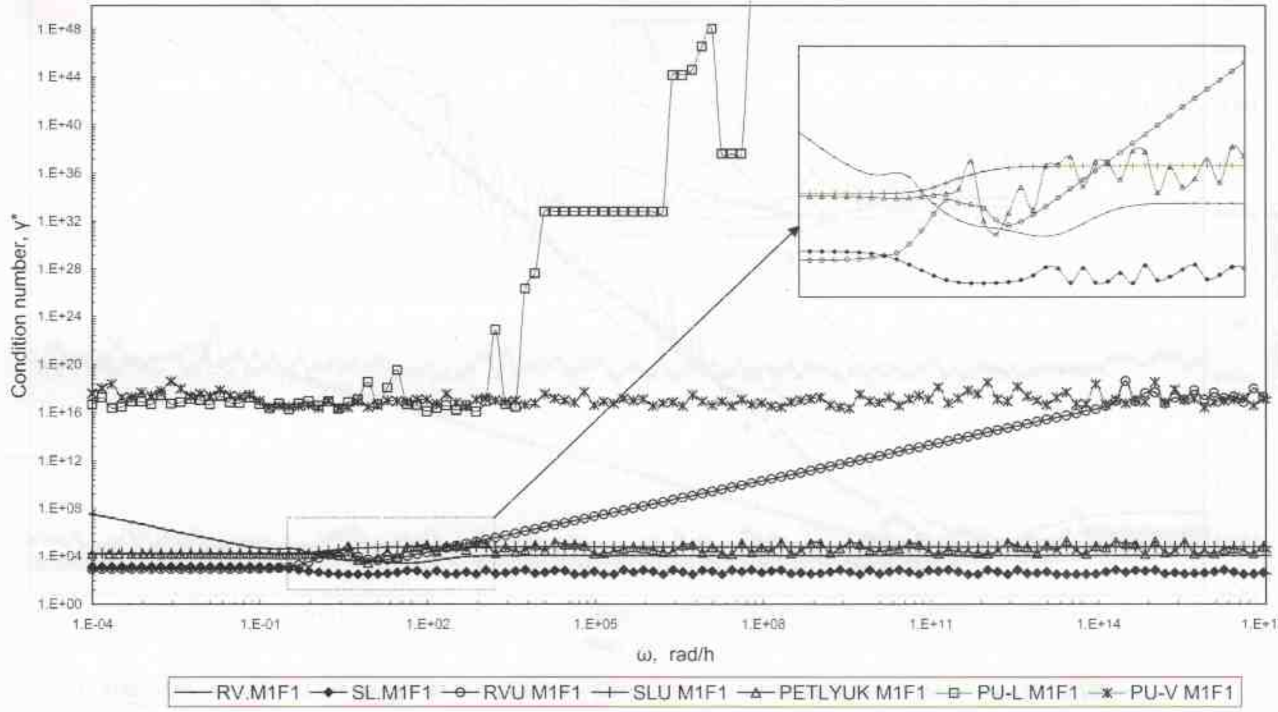


Figure 4. Condition number, case M1F1.

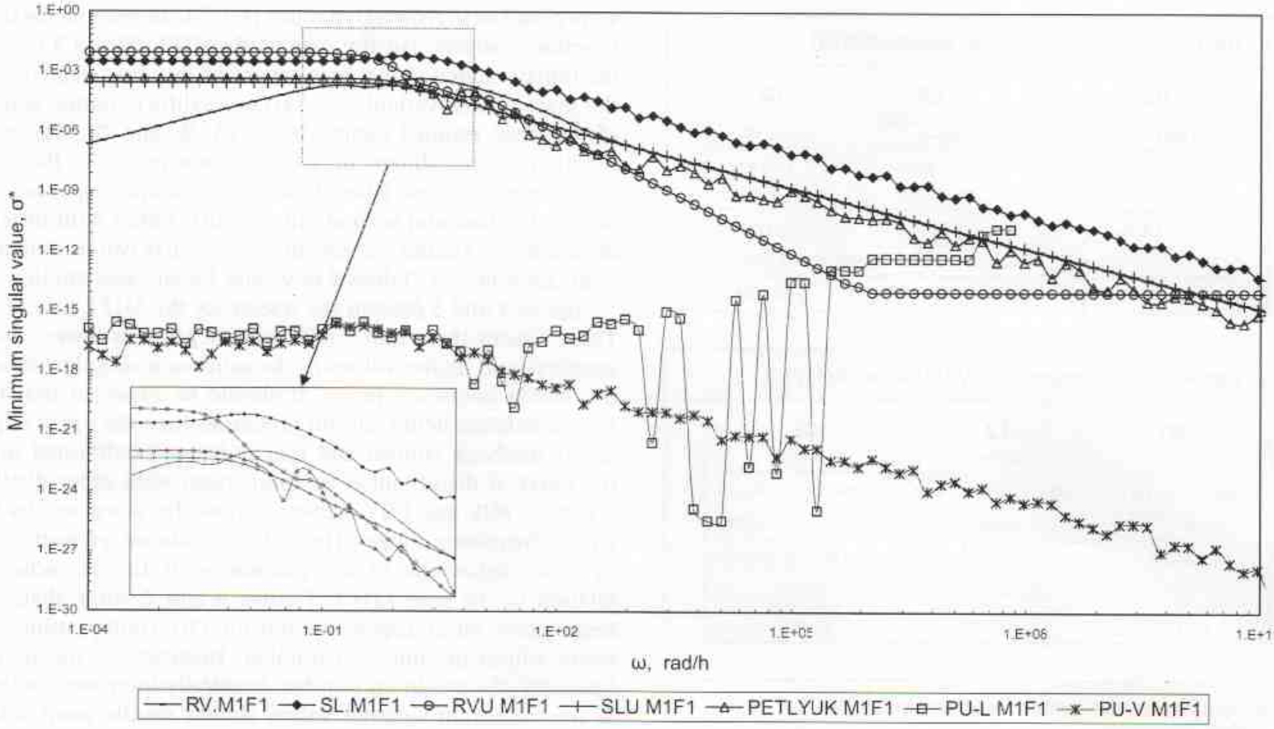


Figure 5. Minimum singular value, case M1F1.

dynamic properties in comparison to SL and RV schemes. In general, it can be concluded that a reduction in the number of recycles provided an improvement of the controllability properties in the Petlyuk-like column structures.

When the distillation sequences were studied for the case M3F1, the SLU configuration presented lower values of the condition number and higher values of σ_* for almost all the frequency ranges. Therefore, the SLU arrangement is expected to require less control efforts under feedback operation and it is better conditioned. According to the SVD results, the Petlyuk column showed good control properties since it presented lower values of the condition number and similar values of the minimum singular value as compared with the SLU arrangement. In the case of M3F1, at higher values of frequency, the PUL and PUV configurations seemed to provide the worst choice because they showed the highest values of the condition number and the lowest σ_* values.

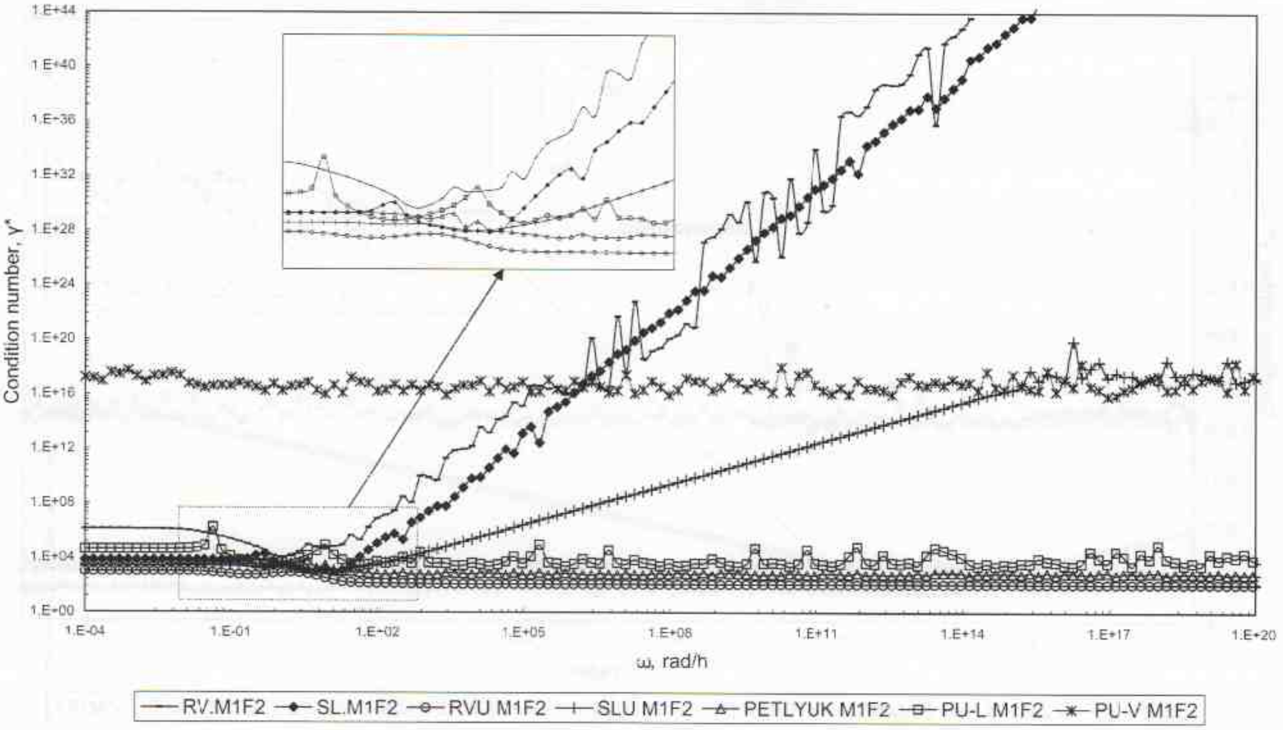


Figure 6. Condition number, case M1F2.

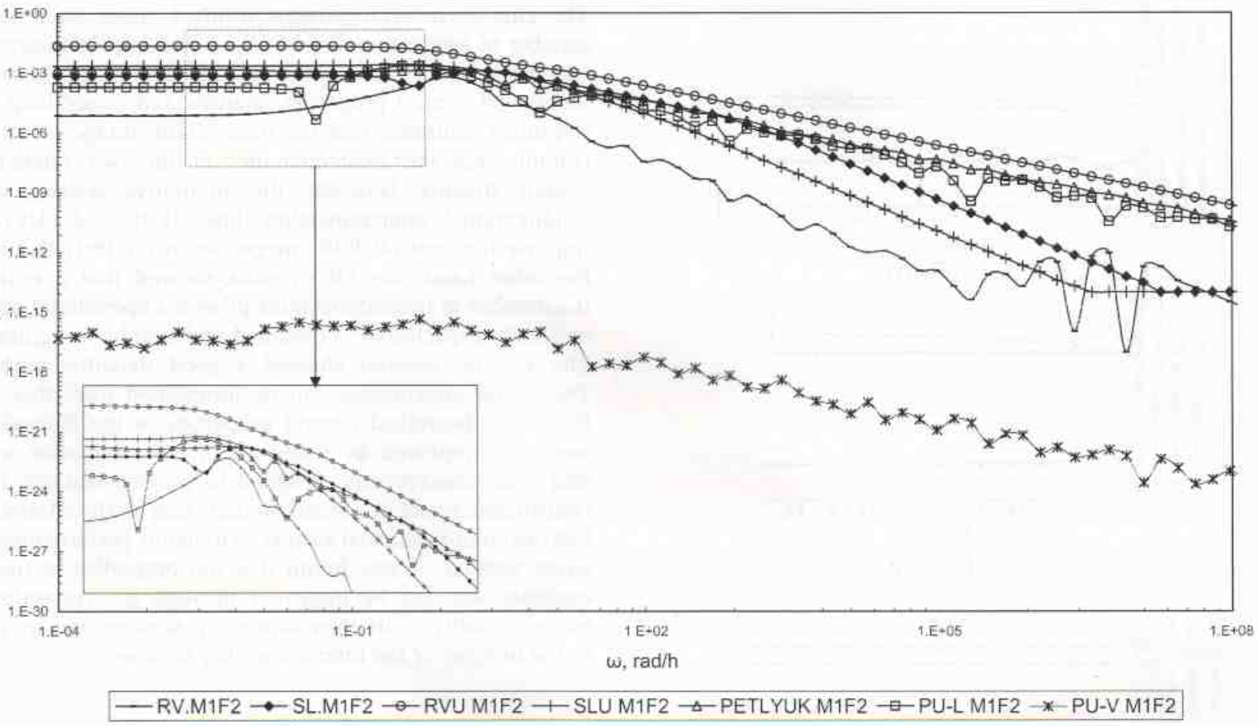
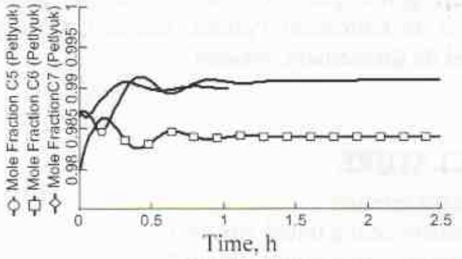
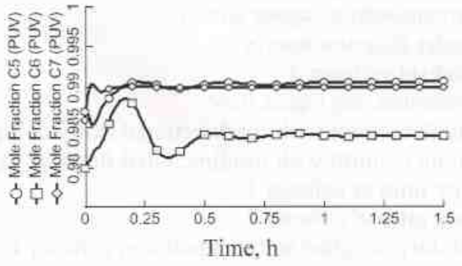


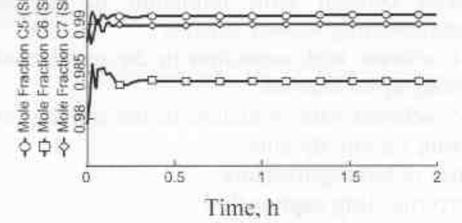
Figure 7. Minimum singular value, case M1F2.



a) Petlyuk



b) PU-V



c) SL

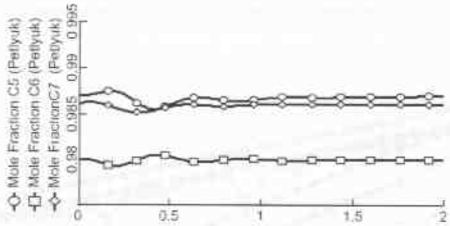
Figure 8. Dynamic responses for set point changes (three closed-loops) in some representative sequences.

This implies large movements in the control valves for changes in the set points and load rejection for the PUL and PUV structures. In general, it can be concluded that the schemes including a reduction in the number of recycles presented better control properties. However, a remark on structures can be established. The results indicated that the Petlyuk configuration presented good control properties in comparison to alternate configurations. The reduction in the number of the recycle streams and the use of bidirectional flows provided an improvement of the controllability properties of the alternate distillation sequences. Finally, the structures including changes in topology and full thermal coupling (PUL and PUV) did not show a good dynamic behaviour.

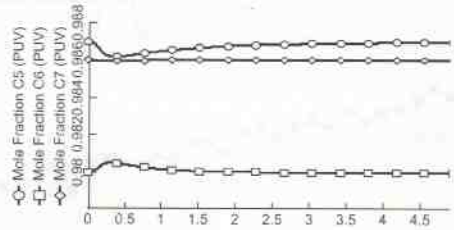
The importance of the SVD technique is the prediction of the closed-loop dynamic behaviour with a small number of open-loop dynamic simulations. To show the consistency between the control properties obtained through the use of the SVD technique and the dynamic responses obtained through rigorous closed-loop dynamic simulations, some dynamic responses can be obtained. In a previous work of Segovia-Hernández et al. (2005b), the closed-loop dynamic simulations of the alternate Petlyuk

Table 10. IAE values for set point changes

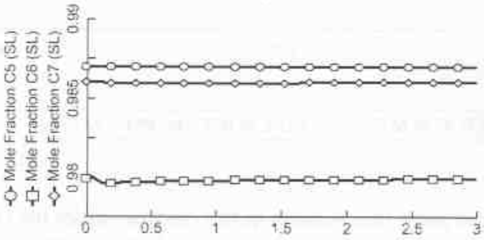
Distillation sequence	Component	IAE
Petlyuk	*n*-Pentane	5.15×10^{-4}
	n-Hexane	2.87×10^{-4}
	n-Heptane	2.35×10^{-4}
PUV	*n*-Pentane	2.02×10^{-4}
	n-Hexane	0.0011
	n-Heptane	6.10×10^{-5}
SL	*n*-Pentane	5.55×10^{-4}
	n-Hexane	7.77×10^{-4}
	n-Heptane	9.20×10^{-4}



a) Petlyuk



b) PU-V



c) SL

Figure 9. Dynamic responses for a disturbance in the composition in the feed in some representative sequences

structures were obtained for SISO controllers. They concluded that the reduction in the number of recycle streams improved the dynamic responses of the alternate Petlyuk structures. For three closed-loops for the case M1F1 in some representative distillation schemes, Figure 8 shows that the SL structure can achieve positive set point changes of 0.005 in each product composition. Also, the Petlyuk column presents good dynamic responses as expected in the SVD analysis. It can be noted that the PUV structure can achieve the set point changes in the compositions, but presents more oscillations in the intermediate component. This result is in agreement with the integral of the absolute error (IAE) values presented in Table 10 for the dynamic responses shown in Figure 9. The PUV structure presents the highest value of the IAE for the dynamic response of the intermediate component.

When the representative schemes were subjected to the effect of a disturbance in the feed compositions (the flow of component B is increased 5% and the flows corresponding to A and C are lowered 2.5% each to keep the total feed flow constant), the three distillation sequences can eliminate the effect of the disturbance, but the SL structure presents less oscillations than the other two complex distillation schemes (Figure 9). As a result, the dynamic responses under closed-loop operation are in agreement with the control properties inferred from the SVD results. Finally, the dynamic responses could be improved by selecting different control variables or pairings in the control loops.

CONCLUSIONS

A comparative study on the theoretical control properties of the Petlyuk column and six alternative schemes has been conducted. The alternative arrangements resulted either by reducing the number of interconnections or by correcting the bidirectionality of the interconnections in the Petlyuk column. The results of the theoretical control properties analysis and closed-loop dynamic responses indicated that the main disadvantage of the Petlyuk column was the bidirectionality of the two interconnecting vapour streams. However, the alternative systems with two unidirectional interconnecting flows (PUL and PUV) did not improve the controllability properties of the Petlyuk system. On the other hand, the SVD results showed that a reduction in the number of interconnections provided operational advantages originally expected for a modified and simpler structural design. The Petlyuk column showed a good dynamic performance. Two major conclusions can be formulated from this research. First, the theoretical control properties of the Petlyuk column were not improved by using all of the alternative sequences that were conceptually designed to provide simpler and more controllable structures. Also, by analyzing all the alternatives, the Petlyuk column showed a superior dynamic performance in many cases. Second, it was found that the properties of the Petlyuk configuration can be improved through the correction of the bidirectionality of its interconnecting streams and by reduction in the number of the interconnecting streams.

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NOMENCLATURE

ABC	ternary mixture
FL1	interconnecting liquid stream 1
FL2	interconnecting liquid stream 2
FV1	interconnecting vapour stream 1
FV2	interconnecting vapour stream 2
G	transfer function matrix
L2	liquid side stream 2
LF	interconnecting liquid flow
PU-L	Petlyuk column with unidirectional flows of liquid
PU-V	Petlyuk column with unidirectional flows of vapour
R1	reflux ratio of column 1
R2	reflux ratio of column 2
C1	heat duty supplied to the reboiler of column 1
C2	heat duty supplied to the reboiler of column 2
RL	Petlyuk column with reduction in the number of interconnecting liquid streams
RV	Petlyuk column with reduction in the number of interconnecting vapour streams
RL-U	PU-L scheme with reduction in the number of interconnecting liquid streams
RV-U	PU-V scheme with reduction in the number of interconnecting vapour streams
V	matrix of left eigenvectors
VF	interconnecting vapour flow
W	matrix of right eigenvectors
x_A	mole fraction of component A
x_B	mole fraction of component B
x_C	mole fraction of component C

Greek Symbols

γ^* condition number
σ^* maximum singular value
σ_* minimum singular value
Σ matrix of singular values
λ_i eigenvalue i
ω frequency

REFERENCES

Abdul-Mutalib, M. I. and R. Smith, "Operation and Control of Dividing Wall Distillation Columns. Part I: Degrees of Freedom and Dynamic Simulation," Trans. Inst. Chem. Eng. **76**, 308 (1998).

Agrawal, R. and A. T. Fidkowski, "More Operable Arrangements of Fully Thermally Coupled Distillation Columns," AIChE J. **44**, 2565 (1998).

Amminudin, K. A., R. Smith, D. Y.-C. Thong and G. P. Towler, "Design and Optimization of Fully Thermally Coupled Distillation Columns. Part I: Preliminary Design and Optimization Methodology," Trans. Inst. Chem. Eng. **79**, 701 (2001).

Chen, J., J. S. Freudenberg and C. N. Nett, "The Role of the Condition Number and the Relative Gain Array in Robustness Analysis," Automatica **30**, 1029 (1994).

Dunnebier, G. and C. Pantelides, "Optimal Design of Thermally Coupled Distillation Columns," Ind. Eng. Chem. Res. **38**, 162 (1999).

Halvorsen, I. J. and S. Skogestad, "Optimal Operation of Petlyuk Distillation: Steady-State Behavior," J. Process Control **9**, 407 (1999).

Hernández, S. and A. Jiménez, "Design of Energy Efficient Petlyuk Systems," Comput. Chem. Eng. **23**, 1005 (1999a).

Hernández, S. and A. Jiménez, "Controllability Analysis of Thermally Coupled Distillation Systems," Ind. Eng. Chem. Res. **38**, 3957 (1999b).

Hernández, S., J. G. Segovia-Hernández and V. Rico-Ramírez, "Thermodynamically Equivalent Distillation Schemes to the Petlyuk Column for Ternary Mixtures," Energy **31**, 1840 (2006).

Jiménez, A., S. Hernández, F. A. Montoy and M. Zavala-García, "Analysis of Control Properties of Conventional and Nonconventional Distillation Sequences," Ind. Eng. Chem. Res. **40**, 3757 (2001).

Jiménez, A., N. Ramírez, A. Castro and S. Hernández, "Design and Energy Performance of Alternative Schemes to the Petlyuk Distillation System," Trans. Inst. Chem. Eng. **81**, 518 (2003).

Kaibel, G. and H. Schoenmarkers, "Process Synthesis and Design in Industrial Practice," in "Proc. ESCAPE-12 (Computer Aided Process Engineering, 10)," J. Grievink and J. V. Schijndel, Eds., Elsevier, Amsterdam (2002), p. 9.

Kaymak, D. B. and W. L. Luyben, "Comparison of Two Types of Two-Temperature Control Structures for Reactive Distillation Columns," Ind. Eng. Chem. Res. **44**, 4625 (2005).

Kim, Y. H., "Structural Design and Operation of a Fully Thermally Coupled Distillation Column," Chem. Eng. J. **85**, 289 (2002).

Kim, Y. H., "An Alternative Structure of a Fully Thermally Coupled Distillation Column for Improved Operability," J. Chem. Eng. Jpn. **36**, 1503 (2003).

Klema, V. C. and A. J. Laub, "The Singular Value Decomposition its Computation and Some Applications," IEEE Trans. Automat. Contr. AC-25, 164 (1980).

Lau, H., J. Álvarez and K. F. Jensen, "Synthesis of Control Structures by Singular Value Analysis: Dynamic Measurements of Sensitivity and Interaction," AIChE J. **31**, 427 (1985).

Morari, M., "Design of Resilient Processing Plants III: A General Framework for the Assessment of Dynamic Resilience," Chem. Eng. Sci. **38**, 1881 (1983).

Muralikrishna, K., K. P. Madhavan and S. S. Shah, "Development of Dividing Wall Distillation Column Design Space for a Specified Separation," Trans. Inst. Chem. Eng. **80**, 155 (2002).

Petlyuk, F. B., V. M. Platonov and D. M. Slavinskii, "Thermodynamically Optimal Method for Separating Multicomponent Mixtures," Int. Chem. Eng. **5**, 555 (1965).

Segovia-Hernández, J. G., S. Hernández and A. Jiménez, "Control Behavior of Thermally Coupled Distillation Sequences," Trans. Inst. Chem. Eng. **80**, 783 (2002).

Segovia-Hernández, J. G., S. Hernández, V. Rico-Ramírez and A. Jiménez, "A Comparison of the Feedback Control Behavior Between Thermally Coupled and Conventional Distillation Schemes," Comput. Chem. Eng. **28**, 811 (2004).

Segovia-Hernández, J. G., S. Hernández, A. Jiménez and R. Femat, "Dynamic Behavior and Control of the Petlyuk Scheme Via a Proportional–Integral Controller with Disturbance Estimation (PII^2)," Chem. Biochem. Eng. Q. **19**, 243 (2005a).

Segovia-Hernández, J. G., S. Hernández and A. Jiménez, "Analysis of Dynamic Properties of Alternative Sequences to the Petlyuk Column," Comput. Chem. Eng. **29**, 1389 (2005b).

Segovia-Hernández, J. G., S. Hernández and A. Jiménez, "A Short Note about Energy-Efficiency Performance of Thermally Coupled Distillation Sequences," Can. J. Chem. Eng. **84**, 139 (2006).

Serra, M., A. Espuña and L. Puigjaner, "Control and Optimization of the Divided Wall Column," Chem. Eng. Process. **38**, 549 (1999).

Skogestad, S. and I. Postlethwaite, "Multivariable Feedback Control: Analysis and Design," 2nd ed., John Wiley and Sons, Hoboken, NJ, U.S.A. (2005).

Tedder, D. W. and D. F. Rudd, "Parametric Studies in Industrial Distillation: Part I. Design Comparisons," AIChE J. **24**, 303 (1978).

Triantafyllou, C. and R. Smith, "The Design and Optimization of Fully Thermally Coupled Distillation Columns," Trans. Inst. Chem. Eng. **70**, 118 (1992).

Wolff, E. A. and S. Skogestad, "Operation of Integrated Three-Product (Petlyuk) Distillation Columns," Ind. Eng. Chem. Res. **34**, 2094 (1995).

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