Dynamic Behavior and Control of the Petlyuk scheme via a Proportional-Integral controller with disturbance estimation (PII^2)

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Original scientific paper
Received: January 21, 2005
Accepted: May 25, 2005

Because of its significant energy savings, the Petlyuk column provides an interesting alternative to the use of conventional distillation systems. Its dynamic and operational characteristics, however, require a proper understanding to promote its practical use. In this work, a proportional-integral controller with dynamic estimation of unknown disturbances is implemented for the control of a Petlyuk column for the separation of ternary mixtures. The proposed controller comprises three feedback terms: proportional, integral and quadratic actions. The last term provides a dynamic estimation of uncertainties and improves the closed-loop performance. Comparison with the classical PI control law was carried out to analyze the performance of the proposed controller in face to unknown feed disturbances and set point changes. The results show that the closed-loop response of the Petlyuk column is significantly improved with the proposed controller.

Key words:
Distillation, thermally coupled distillation, distillation control, disturbance rejection

Introduction

The yearly cost of many chemical processes is noticeably affected by the energy consumption of distillation columns. This factor has influenced the search for alternative designs for distillation systems with lower energy demands than those observed for the conventional distillation columns. One strategy is the use of thermal coupling, in which an interconnection of a liquid stream of one column with a vapor stream from another one achieves two positive effects. One, a heat transfer equipment (condenser or reboiler) is eliminated, and two, with a proper design procedure (particularly with a proper selection of values for the interconnecting flowrates), energy savings with respect to conventional distillation systems can be obtained. For the separation of ternary mixtures, schemes with side columns and an arrangement with a prefractionator followed by a main column with three product streams (known as the Petlyuk column) may show lower energy consumption levels than the conventional direct and indirect distillation trains. Of those coupled arrangements, the Petlyuk column (also known as the fully thermally coupled distillation column, Figure 1) has been shown to provide the highest energy savings. The Petlyuk column had not gained interest in the process industries until recent times (*Hairston*[1]) even though its concept was established some 50 years ago (*Brugma*;[2] *Wright*[3]). Savings in, both, energy consumption and fixed investment can be accomplished through the implementation of such a separation scheme. Theoretical studies have shown that Petlyuk columns can save up to 30 % in energy costs compared to conventional schemes (e.g. *Petlyuk et al*,[4] *Glinos* and *Malone*,[5] *Fidkowski* and *Krolikowski*[6]). Such results have promoted the development of more formal design methods (*Triantafyllou* and *Smith*,[7] *Hernández* and *Jiménez*,[8] *Dünnebier* and *Pantelides*,[9] *Amminudin* et al.,[10] *Muralikrishna* et al.[11]).

To promote a stronger potential for its industrial implementation, a proper understanding of the operation and control properties of the Petlyuk system are needed to complement the energy savings results. Clearly, the expectance that the dynamic properties of Petlyuk columns may cause control difficulties, compared to the rather well-known behavior of the conventional direct and indirect sequences for the separation of ternary mixtures, has been one of the factors that has contributed to their lack of industrial implementation. Recent efforts have been reported towards the understanding of the dynamic properties of the Petlyuk column

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(*Wolff* and *Skogestad*,[12] *Abdul-Mutalib* and *Smith*,[13] *Hernández* and *Jiménez*,[14] *Serra* et al.,[15] *Jiménez* et al.,[16] *Segovia-Hernández* et al.[17], *Jiménez* et al.[18]).

In this work, we analyze the closed-loop behavior of Petlyuk columns when a novel proportional-integral controller with dynamic estimation of unknown disturbances is implemented (*Alvarez-Ramirez* et al.[19]). The performance of the integrated column under such a controller is compared to the dynamic behavior obtained with a traditional proportional-integral controller. The analysis is based on rigorous dynamic simulations, and two cases are considered: (i) set point tracking and (ii) output regulation under load disturbances in the feed mixture.

Design strategy for the Petlyuk column

The design method follows the work by *Hernández* and *Jiménez*,[8] in which a base design is obtained from the tray structure of a conventional distillation system of a prefractionator followed by two binary separation columns. The tray distribution for the integrated system is obtained through a section analogy procedure with respect to the sequence based on conventional columns. The design is then optimized for energy consumption through a search procedure on the two interconnecting streams, F_L and F_V (see Figure 1). Steady state rigorous simulations are then conducted to test the preliminary design. If the design specifications are met, the preliminary design was successful; otherwise, proper arrangements in the design are implemented until the specified product compositions are obtained.

It should be pointed out that the model does not include the effect of pressure on the linking points between the columns. *Agrawal* and *Fidkowski*[20] have discussed the qualitative aspects of the design pressures for each column of the Petlyuk system. In practice, however, the Petlyuk scheme can be implemented as a dividing wall column; in such a case, those pressure considerations are not relevant since the whole separation process take place within a single shell. The arrangement shown in Figure 1, however, is more convenient for modeling purposes.

PI controller with disturbance estimation (PII^2)

The equations for the process controller complement the dynamic model of the Petlyuk column for the closed loop analysis. The controller is a modification of the classical proportional-integral (PI) mode. The PI control is a classical feedback structure that remains interesting for, both, theoretical and practical research in engineering. Here we design a PI-like (output feedback) control with dynamic estimation of uncertainties via quadratic integration actions. The starting point is the identification of the response in each control point in such a manner that a PI-like feedback can be designed. The main idea is to regulate the output of the Petlyuk column according to the control loop scheme shown in Figure 2 (i.e., each effluent component concentration is separately regulated.) The proposed controller computes an estimated value of the input disturbance $d = d(t)$ departing from the

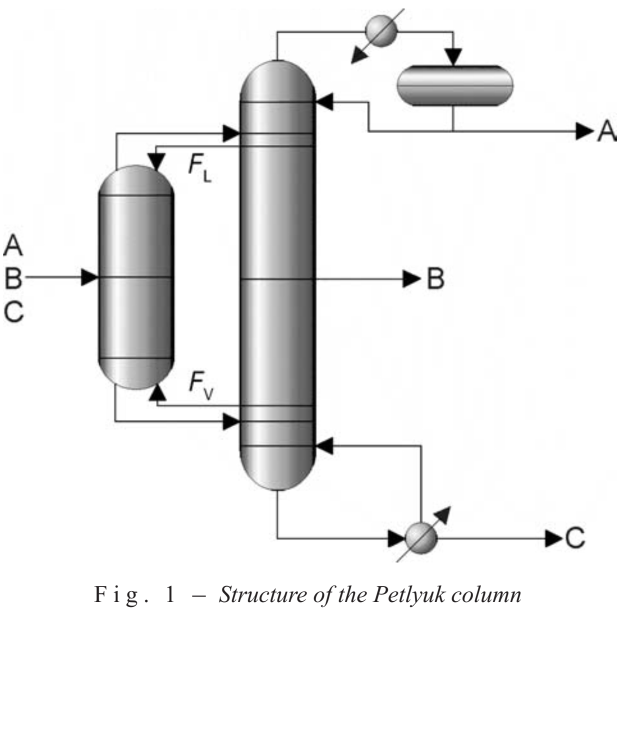


Fig. 1 – *Structure of the Petlyuk column*

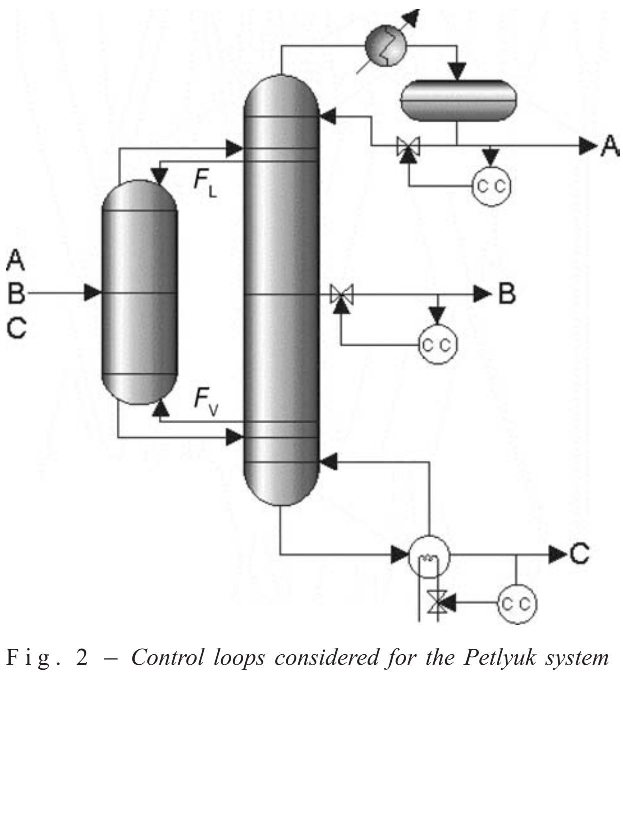


Fig. 2 – *Control loops considered for the Petlyuk system*

output. The source of the disturbance in each control point is the concentration from each tray in the column

The Petlyuk column, designed from the procedure described in Section 2, is identified by using step responses. Thus, the input-output minimal model in each point, can be represented by

$$G_i(s)=\frac{y_i(s)}{(u_i(s)+d_i(s))}=\frac{K_i}{1+\tau_i s} \tag{1}$$

where i = A,B,C indicates the distillation point, K_i, τ_i respectively stand for the gain and time constant of the plant; d_i denotes the disturbance entering the point i, and is related to the effect of the control loops in the other distillation points. Finally, u_i's are the control inputs, and stand for the reflux flowrate, the side stream flowrate and the reboiler heat duty. The transfer function (1) can be written in the time domain as the following model,

$$\dot{y}_i=-\frac{1}{\tau_i}y_i+\frac{K_{i,p}}{\tau_i}u_i(t)-\frac{K_{i,p}}{\tau_i}d_i(t) \tag{2}$$

where $d(t)$ is a smooth unknown time function. If one considers that the evolution of the disturbance $d(t)$ can be described by the time derivative, the system (2) can be rewritten as the following equivalent system,

$$\begin{aligned}\dot{y}_i&=-\frac{1}{\tau_i}y_i+\frac{K_{i,p}}{\tau_i}u_i(t)-\frac{K_{i,p}}{\tau_i}d\\ \dot{d}&=f(t)\end{aligned} \tag{3}$$

where $f(t)$ is a continuous (possibly nonlinear) uncertain function. Following the ideas by *Femat* et al.[21], the uncertain disturbance d can be estimated via the high-gain observer:

$$\dot{\bar{y}}=-\frac{y}{\tau}+K_{\mathrm{P}}u+\bar{d}+g_1(y-\bar{y}) \tag{4}$$

$$\dot{\bar{d}}=g_2(y-\bar{y}) \tag{5}$$

where $\bar{d}$ and $\bar{y}$ are estimated values for d and y, whereas g_1 and g_2 are estimator parameters which must be strictly positive to guarantee the stability of the estimation errors $e_1=(y-\bar{y})$ and $e_2=(d-\bar{d})$ to the origin, i.e., if g_1, $g_2>0$ then $(e_1,e_2)\rightarrow(0,0)$ for all time $t>t_0>0$ and any initial condition $e_{1,0}=e_1(0)$, $e_{2,0}=e_2(0)\Rightarrow y\rightarrow\bar{y}$ and $d\rightarrow\bar{d}$ for all $t>t_0>0$ and any initial condition at the physically realizable operation of the column (*Femat* et al.[21]). The dynamics of $\bar{d}$ are driven by the estimation error $(y-\bar{y})$, which is supposed to reflect the deviation between d and $\bar{d}$.

Consider now the feedback interconnection as in Figure 2. The classical PI control can be written as the following dynamic system (*Luyben*[22]),

$$\begin{aligned}u&=K_{\mathrm{C}}(y-r)-z\\ z&=K_1(y-r)\end{aligned} \tag{6}$$

where r stands for the set point, y denotes the system output and z is the integral of the control error. The constants K_{C} and $K_{\mathrm{I}}=K_{\mathrm{C}}/\tau_i$ (where τ_i denotes the reset time) stand for the proportional and integral gains, respectively.

Through the combination of the observer for uncertain disturbances (4) – (5) and the PI controller (6), a PI-like structure is obtained that is robust against disturbances $d(t)$ because, if the system (2) provides an estimate of d, it can be used to counteract the effects of the disturbance in the feedback loop. Thus, the following feedback controller can be proposed (*Alvarez-Ramírez* et al.[19]),

$$u=\frac{\left\lfloor\left(\frac{1}{\tau}-\frac{1}{\tau_{\mathrm{c}}}\right)y-\bar{d}\right\rfloor}{K_{\mathrm{P}}} \tag{7}$$

where $\bar{d}$ is obtained from (5). Equations (4) to (7) comprise the PI-like controller with second-order compensation of uncertainties. Note that, if the disturbance $d(t)$ is exactly known, then the feedback control (7) decouples the effects of the disturbance $d(t)$ and the reference signal and, as a consequence, the classical PI feedback is obtained.

Since the proposed feedback is linear, from simple manipulations we can obtain a transfer function for the controller:

$$C(s)=K_{\mathrm{C}}\left[1+\frac{1}{\tau_1 s}+\frac{K_{\mathrm{e}}}{s(s+g_1)}\right] \tag{8}$$

where:

$$K_{\mathrm{C}}=-\frac{\frac{1}{\tau}-\frac{1}{\tau_{\mathrm{C}}}}{K_{\mathrm{P}}} \tag{8a}$$

$$\tau_1^{-1}=-\frac{g_2}{\frac{1}{\tau}-\frac{1}{\tau_{\mathrm{C}}}} \tag{8b}$$

$$K_{\mathrm{e}}=-g_2\frac{g_1-\frac{1}{\tau_{\mathrm{C}}}}{\frac{1}{\tau}-\frac{1}{\tau_{\mathrm{C}}}} \tag{8c}$$

In summary, the controller (8) is composed of three parts: (i) a proportional feedback, (ii) an integral action $1/\tau_i s$, and (iii) a "second – order" integral action $Kc/s(s + g_1)$. The third part enhances the disturbance estimation capabilities of the PII^2. Such a quadratic term provides a dynamic estimation of the input d, which can represent load disturbances (regulation problem) or step changes in references (servo-control problem).

The following useful parameterization of the estimator gains g_1 and g_2 allows us to guarantee asymptotic stability to the desired reference (*Femat* et al.[21]): $g_1 = 2L$ and $g_2 = L^2$, where $L > 0$ is arbitrarily chosen. The number of controller parameters is only two: K_c and L. The parameter L is interpreted as the rate of the estimated value convergence. For further details on tuning and closed-loop stability analysis of the PII^2 controller, see *Alvarez-Ramirez* et al.[19] and *Femat* et al.[21], respectively.

Closed-loop implementation

The implementation of the output feedback control for the distillation column can be configured such that only the liquid composition of the output flowrate is regulated (i.e., uncoupled one-point configuration control) as in *Jiménez* et al.[16], and *Segovia-Hernández* et al.[23]. In such a configuration, the liquid compositions for the main product streams A, B and C (see Figure 2) were taken as the controlled variables whereas, respectively, the reflux flowrate, the side stream flowrate and the reboiler heat duty were chosen as the manipulated variables. The ideas behind the simulations are (i) to show that the Petlyuk column can be controlled by exploiting a simple control configuration and (ii) to improve the closed-loop performance by implementing a proportional-integral feedback with dynamic estimation of unknown disturbances. The performance of the PII^2 controller was compared with the performance of the classical PI control action, which is a widely-used type of controller in the chemical industry. Both types of controllers were tuned following the integral of the absolute error (IAE) criterion. Therefore, a set point change was implemented for each control loop, and the values of the control gains (K_C and τ_i in the case of PI controllers, or K_C and L in the case of the PII^2) that provided a minimum value of the IAE were detected. The closed loop tests were then conducted. It is important to say that the pairings in the control loops, were chosen according to practical issues and previous works in the control of thermally coupled distillation sequences (*Cárdenas* et. al.[24]). The composition of the distillate was controlled by manipulating the reflux rate, the bottoms composition was tied to the heat duty supplied to the reboiler and the side product was adjusted by manipulating the flow rate of the sidestream.

Case studies

Three hydrocarbon mixtures were considered for the case studies: n-pentane, n-hexane and n-heptane (mixture 1), n-butane, isopentane and n-pentane (mixture 2), and isobutane, n-butane and n-hexane (mixture 3). The molar composition in the feed was taken as 0.40, 0.20, 0.40 in A, B and C, respectively; such a molar composition reflects typical values for which significant energy savings have been reported. The feed flowrate was taken as 45.4 kmol/hr as saturated liquid, and the specified purities for the product streams were assumed as 98.7, 98 and 98.6 % for A, B and C, respectively.

Two sets of dynamic simulations were carried out. (i) *Servo-control*: A step change was induced in the set point for each product composition under SISO feedback control at each output flowrate (see Figure 2), and (ii) *Regulation under load disturbances*: feed composition disturbances were induced. A 5 % change in the composition of one component (with a proportional adjustment in the composition of the other components to keep the same total feed flowrate) was implemented as a feed disturbance for each test.

The simulations were carried out assuming first SISO operation for the control of each individual component. The problem of controlling the composition of the three product streams simultaneously finished the dynamic tests (MIMO). The tuning in the MIMO problem is clearly more complicated as compared to the SISO case. As a result, we followed a sequential tuning procedure. The loop for the control of component A was tuned first, and then the control loop for component B was tuned with the loop for component A on automatic, and finally the control loop for component C was tuned with control loops A and B on automatic.

Dynamic responses with one closed loop

The response of each individual product stream to the servo and load disturbance problem was obtained. For each case, the products not being analyzed were assumed to be under open loop operation. All dynamic simulations were performed after a base design for the Petlyuk column and the minimum energy consumption (which provided therefore the desired operating point) was obtained.

Light component

Figure 3 shows the dynamic responses for product stream A for both types of tests conducted. When a feed disturbance was implemented, both, the PI and the PII^2 controllers successfully rejected such an effect to bring the product composition back to its design value. However, the response of the Petlyuk system is remarkably improved through the use of the PII^2 controller (see Figure 3a). The dynamic action of the control valve is displayed in Figure 3b. It was assumed that the control valve was originally at 50 % opening for the operation under the specified design conditions. In agreement to the smooth disturbance rejection of the PII^2 controller, the control valve shows a quick adjustment towards the new steady state of the manipulated variable, which may also be interpreted as a lower control effort.

When a set point change in the required composition of product A was implemented (a change from 0.987 to 0.991), the new controller reaches the new steady state faster than the conventional PI controller (Figure 3c). The corresponding action of the control valve can be observed in Figure 3d.

The numerical values of the IAE for the disturbance rejection cases are given in Table 1. The superior behavior of the PII^2 controller for mixture 1 is reflected in the corresponding IAE values. When mixtures 2 and 3 were considered, the behavior of the system under the new controller was slightly better for mixture 2, and significantly better for mixture 3, as indicated by the IAE values of Table 1. This behavior of the PII^2 controller is induced by the disturbance estimator, which resembles the structure of linear state observers.

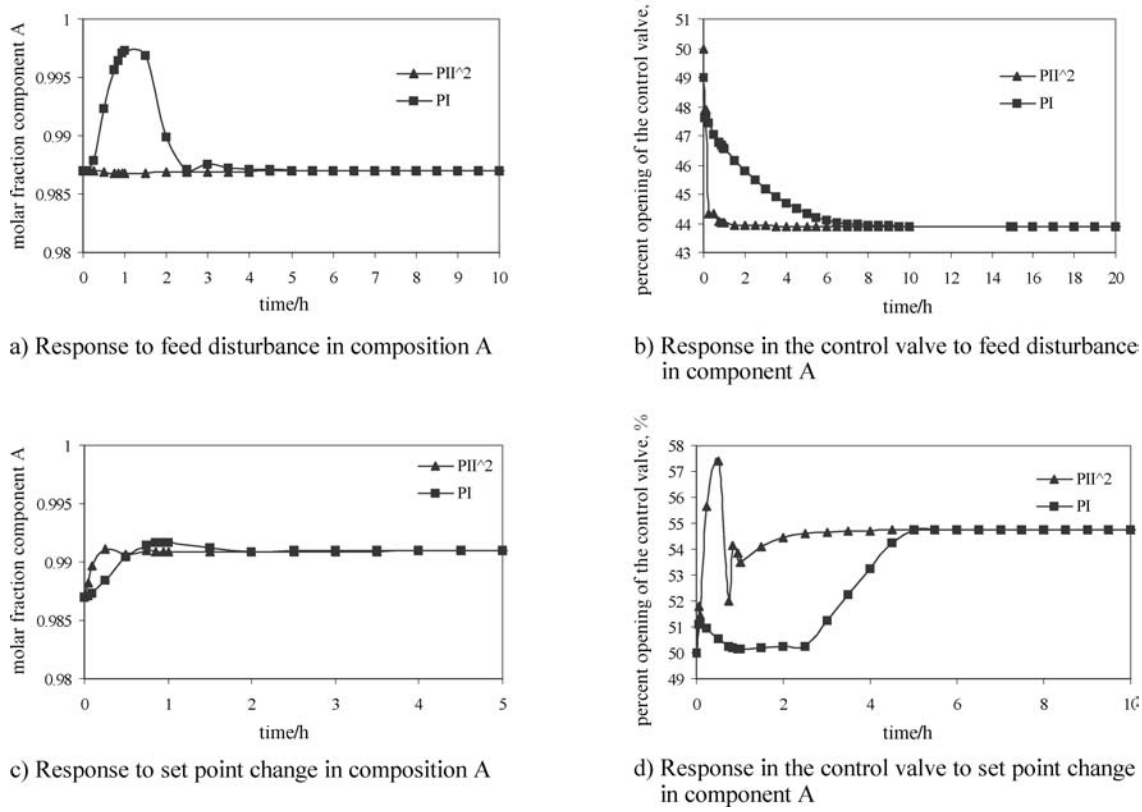
T a b l e 1 – *IAE results for feed disturbance test, component A (one closed loop)*

Mixture	PII^2	PI
1	1.7599×10^{-2}	6.2431×10^{-2}
2	7.575911×10^{-2}	7.726951×10^{-2}
3	5.0428×10^{-2}	0.1290607

Heavy component

For the servo problem a set point change for component C from 0.986 to 0.99 was induced. Figure 4c shows the closed loop response of the heavy component; it can be observed how the PII^2 controller provides a quicker settling time. The control valve action is displayed in Figure 4d.

When a 5 % change in the feed composition of component C was implemented, the responses of



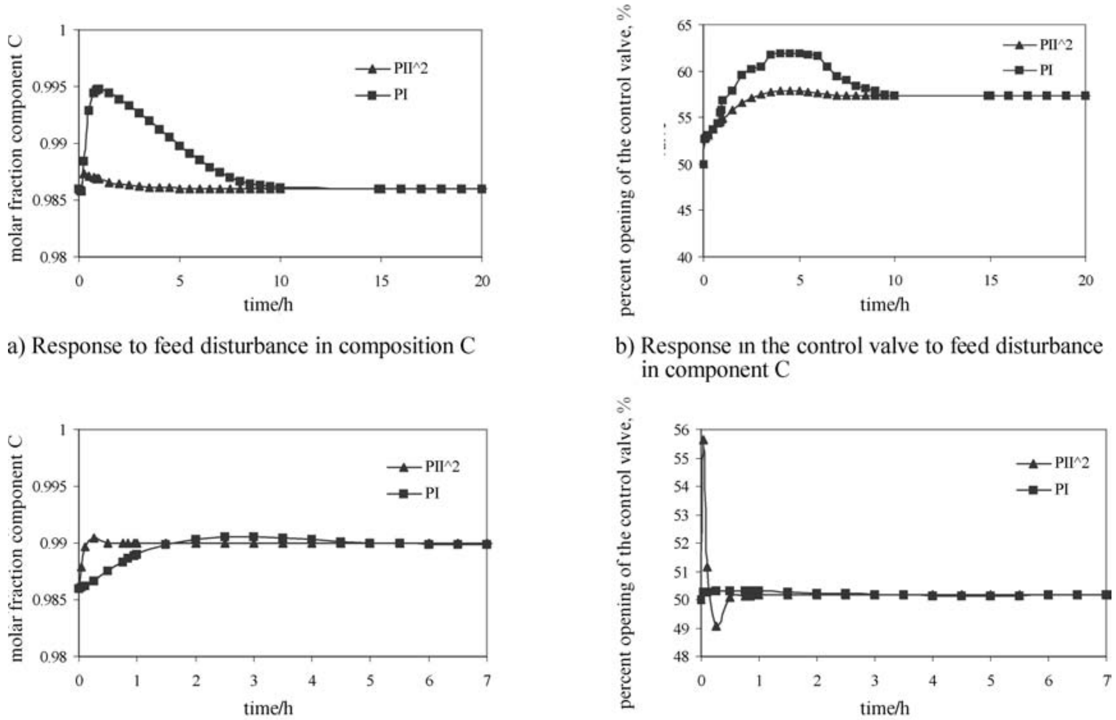
a) Response to feed disturbance in composition A

b) Response in the control valve to feed disturbance in component A

c) Response to set point change in composition A

d) Response in the control valve to set point change in component A

F i g . 3 – *Dynamic responses for component A, mixture 1, one closed loop*



a) Response to feed disturbance in composition C

b) Response in the control valve to feed disturbance in component C

c) Response to set point change in composition C

d) Response in the control valve to set point change in component C

F i g . 4 – *Dynamic responses for component C, mixture 1, one closed loop*

the column for the product stream with the heavy component shown in Figure 4a were obtained. The PII^2 controller provides a smooth and quick adjustment, while the PI controller showed a rather poor behavior, with a high settling time. The control valve motion for each test is reported in Figure 4b; one may notice the higher control effort by the PI controller.

Table 2 shows the values obtained for IAE, when the disturbance tests in the feed composition of component C were implemented for each of the three feed mixtures considered. For the three cases, the action of the PII^2 controller provides a better response than the PI controller, with a remarkable improvement observed for mixture 1. It should be mentioned that when mixtures 2 and 3 were analyzed for feed disturbance rejection, the control valves became at times saturated or closed under the action of the PI controller.

Intermediate component

For the case of the dynamic analysis of the intermediate component, the results of the tests for mixture 1 are displayed in Figure 5. For the servo problem, the change implemented in the set point was from 0.98 to 0.984. Figure 5c shows that the Petlyuk column reaches the new steady state faster under the action of the PII^2 controller. The associated control efforts can be observed in Figure 5d.

When the feed disturbance was implemented, the PII^2 controller provided a significant improvement in the response of the Petlyuk column with respect to the action of the PI controller (see Figure 5a). The opening changes of the control valves can be seen in Figure 5b.

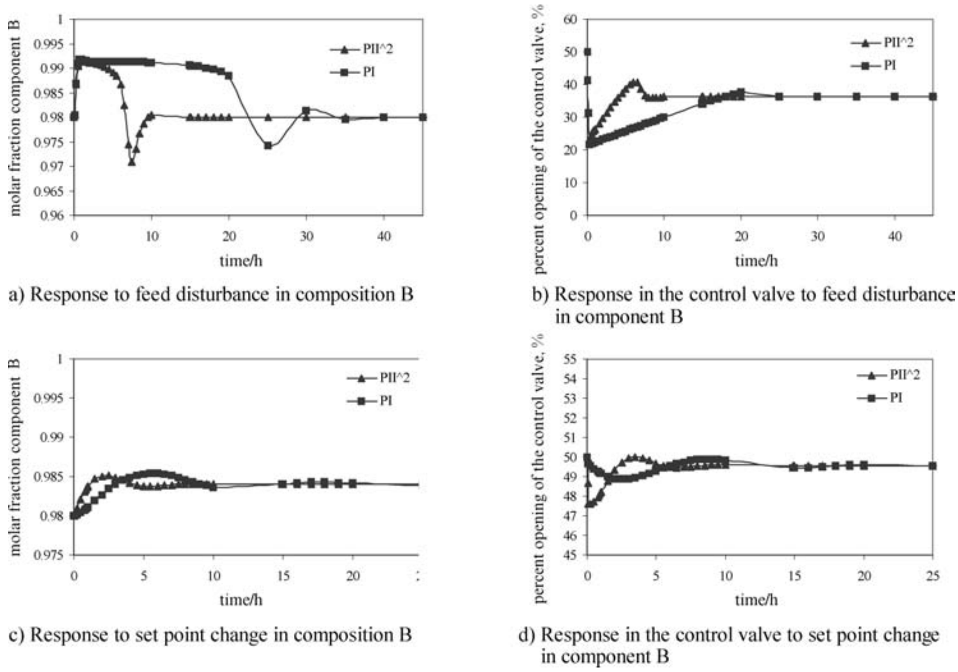
IAE values for the effect of feed composition disturbance of component B for each mixture analyzed are given in Table 3. As reflected by the IAE

T a b l e 2 – *IAE results for feed disturbance test, component C (one closed loop)*

Mixture	PII^2	PI
1	1.3775×10^{-2}	7.1657×10^{-2}
2	5.317069×10^{-2}	6.9009×10^{-2}
3	0.1208054	0.176711

T a b l e 3 – *IAE results for feed disturbance test, component B (one closed loop)*

Mixture	PII^2	PI
1	4.18149×10^{-1}	9.987×10^{-1}
2	6.4794×10^{-2}	4.0099×10^{-1}
3	1.4799×10^{-2}	8.4748×10^{-2}



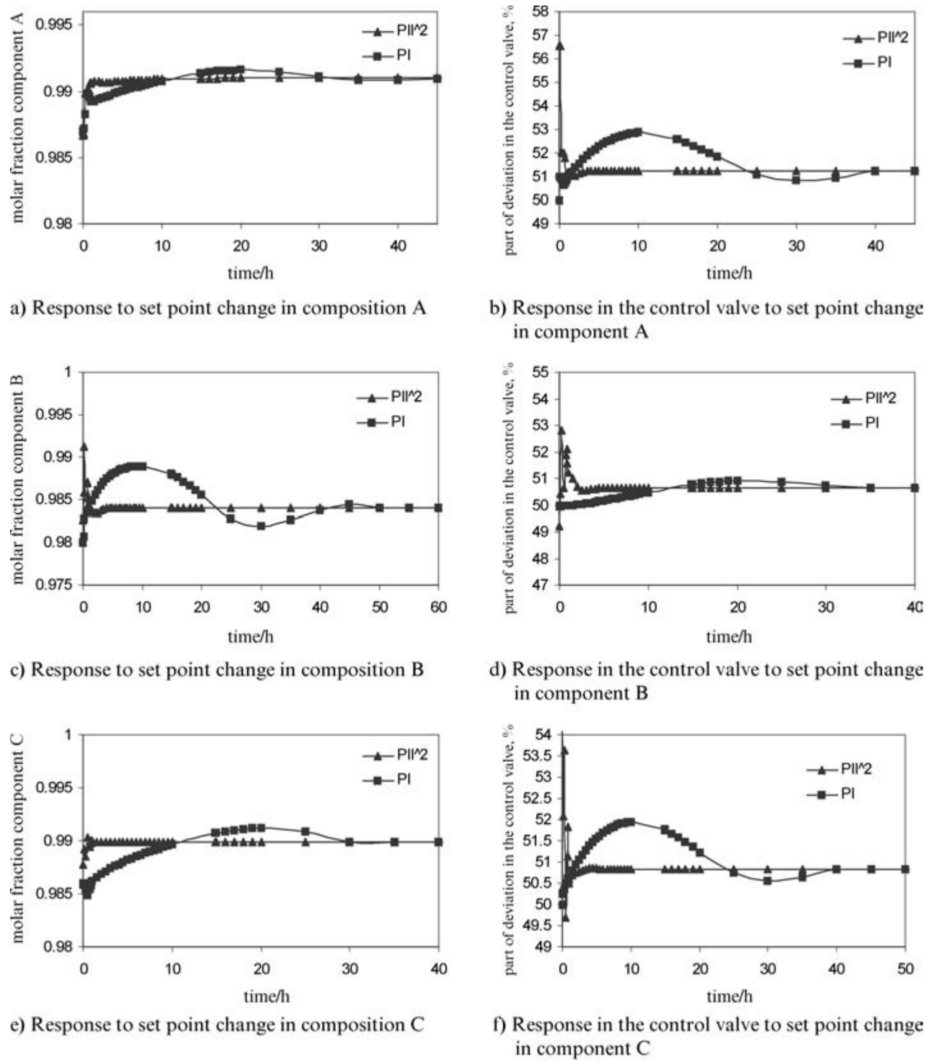
a) Response to feed disturbance in composition B

b) Response in the control valve to feed disturbance in component B

c) Response to set point change in composition B

d) Response in the control valve to set point change in component B

F i g . 5 – *Dynamic responses for component B, mixture 1, one closed loop*



a) Response to set point change in composition A

b) Response in the control valve to set point change in component A

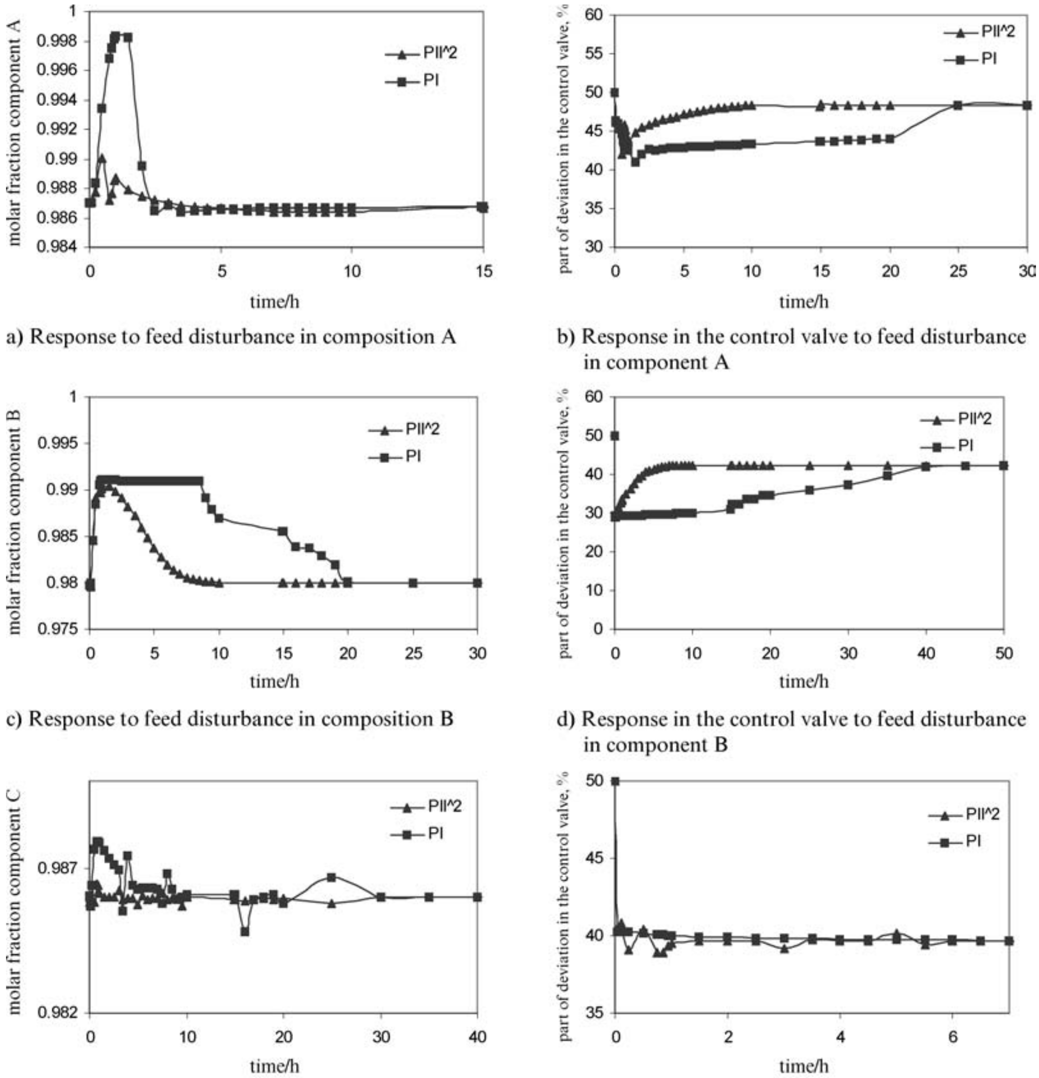
c) Response to set point change in composition B

d) Response in the control valve to set point change in component B

e) Response to set point change in composition C

f) Response in the control valve to set point change in component C

F i g . 6 – *Some dynamic responses for the servo problem with three closed loops*



a) Response to feed disturbance in composition A

b) Response in the control valve to feed disturbance in component A

c) Response to feed disturbance in composition B

d) Response in the control valve to feed disturbance in component B

e) Response to feed disturbance in composition C

f) Response in the control valve to feed disturbance in component C

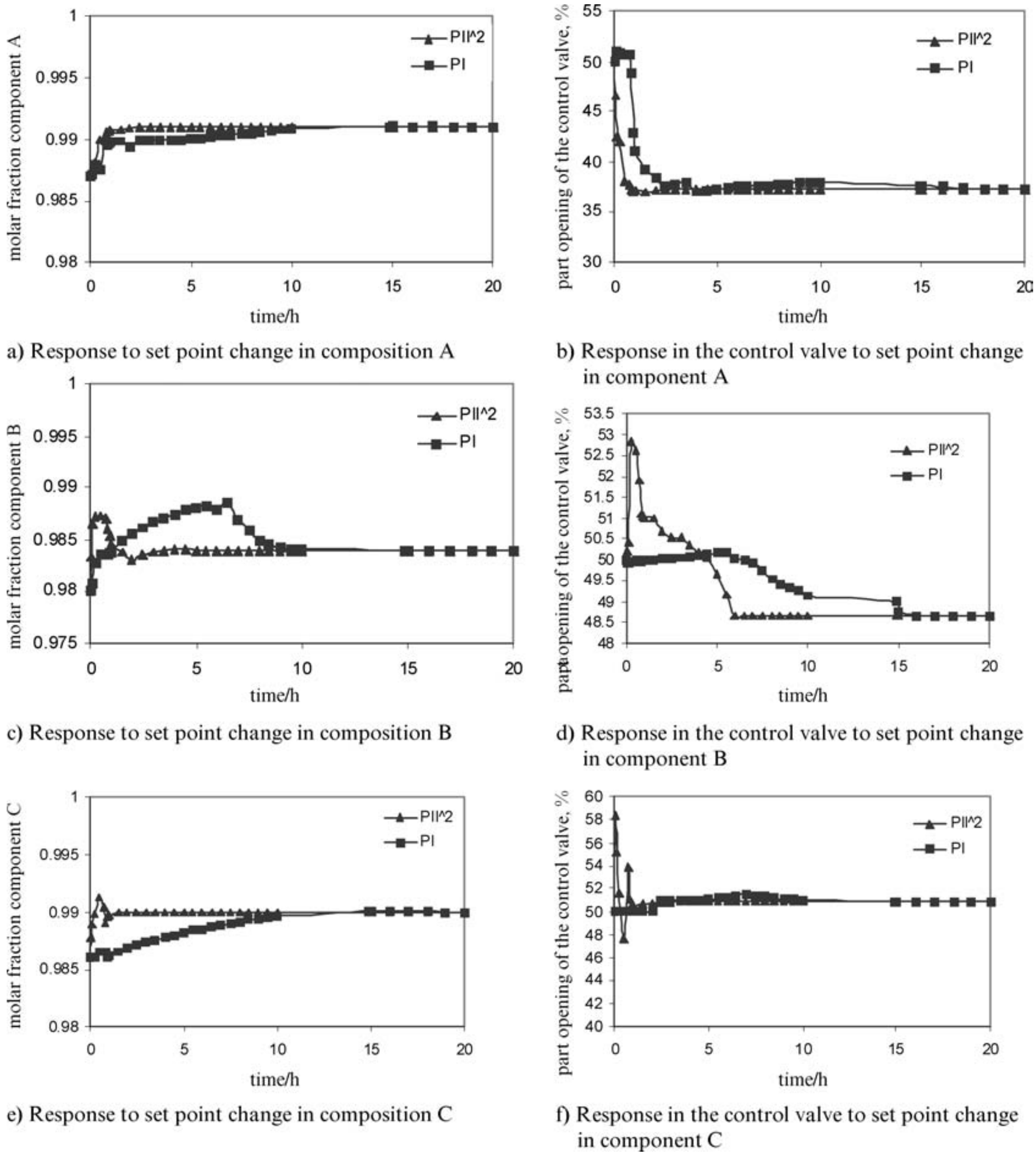
F i g . 7 – *Some dynamic responses for the feed disturbance problem with three closed loops*

values, the implementation of the PII^2 controller shows a significant improvement for disturbance rejection over the responses provided by the traditional PI controller.

Dynamic responses with three closed loops

A final test on the dynamic responses of the Petlyuk column with the three control loops closed was made, both, for set point tracking and feed disturbance rejection analysis. The responses for set point tracking test for each control loop, along with the corresponding control valve motions, are shown in Figure 6. In general, the action of the PII^2 controller provides a faster and smoother dynamic response towards the new steady state than the PI controller. One may also notice the lower control efforts (changes in control valves opening) provided by the PII^2 controller.

When the response of the column to feed disturbances was analyzed, the PII^2 controller provided a remarkable improvement over the use of the PI controller. Figure 7 shows some of the responses observed (in particular, the response of the main product stream under a disturbance, induced by a change in composition of that component in the feed stream.) While in several cases the implementation of the PI controller yielded extremely high settling times, the PII^2 controller showed an excellent capability to eliminate the feed disturbance faster and without overshoot problems. As far as control efforts are concerned, the implementation of the PII^2 controller provided in general smoother control actions. It should be noted again that in several of the tests conducted, the control valves became saturated (or completely closed) under the action of PI controllers. When mixtures 2 and 3 were subjected to the same tests, similar trends on the dy-



F i g . 8 – *Dynamic responses for mixture 2 for changes in the three set points*

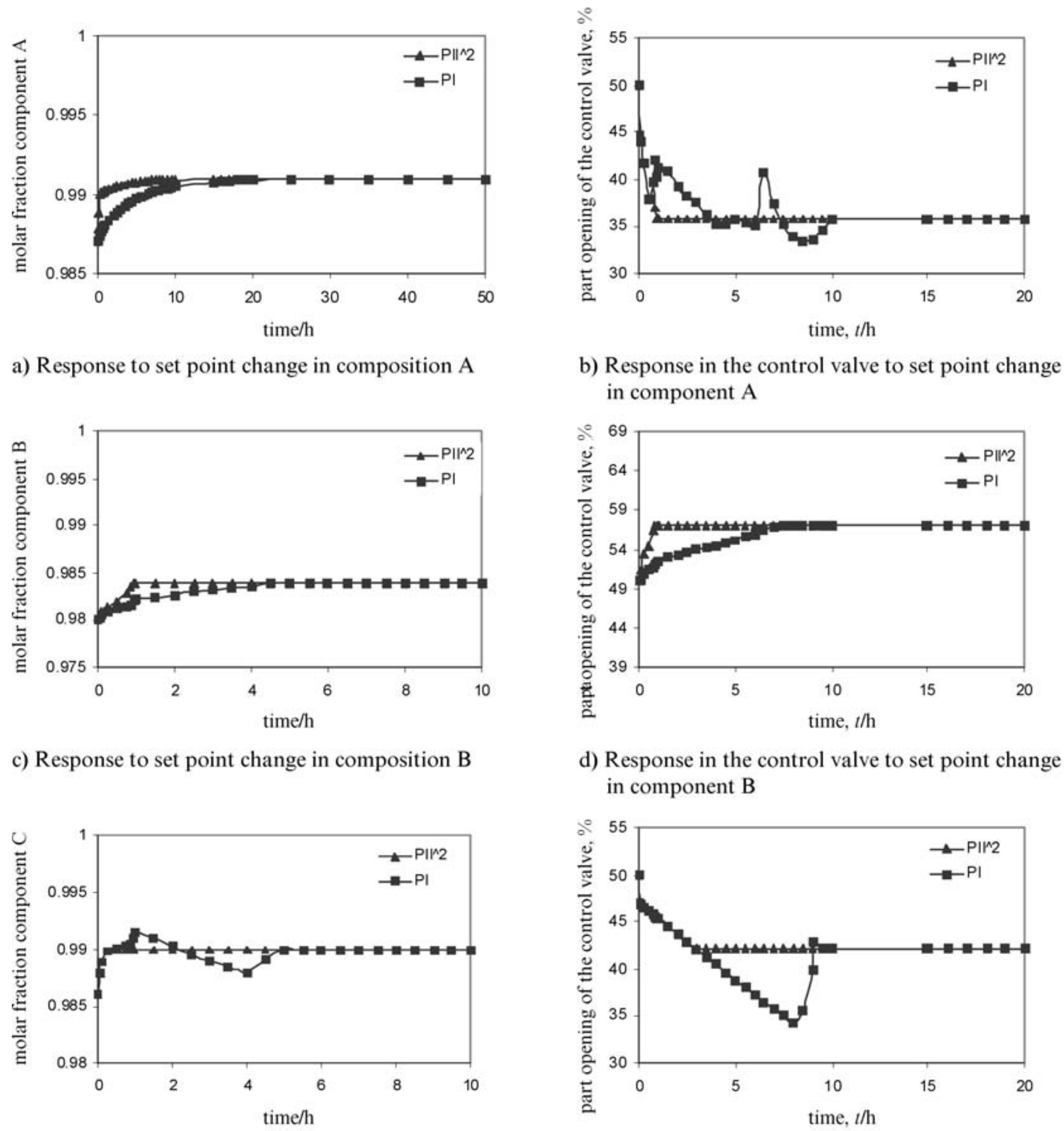
namic responses of the Petlyuk column were obtained. Typical dynamic responses for mixtures 2 and 3 are shown in Figures 8 and 9. Particularly, the PII^2 controller provided a remarkable performance when load changes in the feed composition were considered, as can be noted by the improvement of the IAE values given in Table 4 with respect to the implementation of PI controllers.

Conclusions

A proportional-integral controller with dynamic estimation of uncertainties (PII^2) was implemented in a Petlyuk column. The responses of the separation system were compared to the ones provided by the use of a conventional proportional-in-

T a b l e 4 – *IAE results for feed disturbance test with three closed loops*

Mixture	Component	PII^2	PI
1	A	1.2296×10^{-2}	2.2082×10^{-2}
	B	2.8259×10^{-1}	8.7463×10^{-1}
	C	1.745939×10^{-2}	3.1119×10^{-2}
2	A	8.7427×10^{-2}	7.0042×10^{-2}
	B	1.9947×10^{-1}	6.1824×10^{-1}
	C	6.54001×10^{-2}	3.04974×10^{-1}
3	A	9.0065×10^{-2}	4.788×10^{-1}
	B	7.479×10^{-2}	5.9990×10^{-1}
	C	2.23×10^{-2}	2.004×10^{-1}



a) Response to set point change in composition A

b) Response in the control valve to set point change in component A

c) Response to set point change in composition B

d) Response in the control valve to set point change in component B

e) Response to set point change in composition C

f) Response in the control valve to set point change in component C

F i g . 9 – *Dynamic responses for mixture 3 for changes in the three set points*

tegral (PI) controller. After the parameters for each type of controller were optimized through the minimization of the integral of the absolute error criterion, servo problems and load disturbances in the feed composition were considered. The action of the PII^2 controller provided in both types of tests better closed loop responses, although its effectiveness was more remarkable when the column was subjected to feed disturbances. The properties of the PII^2 controller allow a proper detection of disturbances, such that corrective actions are taken to prevent the controlled output from significant deviations from the desired operation point. The results of this work show that the PII^2 controller provides an excellent potential for the control of the Petlyuk distillation column.

ACKNOWLEDGEMENT

Financial support from PROMEP, México, is gratefully acknowledged.

Nomenclature

d – load disturbance

G – transfer function

g_1, g_2 – uncertainty estimator gains

K_C – PI controller gain, % / %

K_e – PII^2 controller gain, % / %

K_P – process gain, % / %

r – set point

s – Laplace domain variable

u – manipulated input
y – controlled output
z – integral of the control error
F_L – molar flow rate of liquid
F_V – molar flow rate of vapor

Greek letters

τ – process characteristic time, min
τ_C – nominal closed-loop characteristic time, min
τ_I – PI integral reset time, min

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